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An anisotropic viscoelastoplastic model for composites—sensitivity analysis and parameter estimation

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Abstract

Because of their potential in achieving many performance enhancements, composite material systems (e.g. fiber-reinforced composites) are presently called upon to operate under wide range of stresses, temperatures, and loading rates. This in turn requires the development of general material models to capture the significant effects of anisotropy on both elastic and inelastic responses. The starting point in the present contribution is the development of a class of such viscoplastic models. Furthermore, a number of robust, computationally efficient, algorithms are also presented for the development of an overall strategy to estimate the material parameters characterizing these complex models; i.e. rate-dependent plastic flow, non-linear kinematic hardening, thermal/static recovery, anisotropic viscoelastic and viscoplastic flow. The entire procedure is automated through an integrated software namely, COnstitutive Material PARameter Estimator, COMPARE, to enable the determination of an 'optimum' set of material parameters by minimizing the errors between the experimental test data and the predicted response. The key ingredients of COMPARE are (i) primal analysis, (ii) sensitivity analysis, (iii) a gradient-based optimization problem and a (iv) graphical user interface. The estimation of the material parameters is cast as a minimum-error, weighted multi-objective, non-linear optimization problem with constraints. Detailed derivations of the direct differentiation sensitivity expressions are presented. In addition, numerical comparisons of the sensitivities obtained by the more traditional finite difference approaches are given to assess accuracy. Results generated by applying the developed algorithms for anisotropic, strain-controlled tensile (with comparison to typical experimental data) and constant-stress creep tests are presented to demonstrate the ability of the present models to accurately capture time-dependent anisotropic material behavior.

Keywords: A. Polymer–matrix composites; B. Anisotropy; C. Computational modeling; Material characterization

1. Introduction

1.1. General

The use of advanced materials, for example, metals, polymers and ceramics, are at the forefront of today's research. As a result, numerous computational models for predicting both deformation and life for these materials are under development. A key to the effective use of these advanced analysis techniques are accurate and computationally efficient constitutive models. These models must account for both *reversible* and *irreversible* time-dependent deformation. For example, the irreversible time-dependent response component becomes dominate for metals at high temperatures. On the other hand, polymers and rubbers have predominately a purely reversible viscous response.

Significant progress has been made over the years in the development of theories for the phenomenological representations of the time-dependent viscoelastic and viscoplastic constitutive properties. In particular, at least considering the isotropic response case, the mathematical modeling of metal viscoplasticity is presently very well developed, based on the so-called *internal variable* formulism in the thermodynamics of irreversible processes [3,4,9,11,12,17,19]. A number of specialized forms of these modern *unified* viscoplastic models (e.g. isotropic fully associative and non-associative, isothermal or non-isothermal, etc.) have been successfully applied to different metals and other material systems [4,9,12,17]. In addition, several

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attempts have also been made to account for material anisotropy in composites' inelasticity [13,21,22,24]. Here, we utilize, as a starting point, an extended formulation of the anisotropic viscoplastic model reported earlier in Ref. [24], with emphasis on two major enhancements. Firstly, adopting the notion of multiplicity of deformation mechanisms [28], we combine an aggregate of tensorial state variables to account for wider ranges of material relaxation spectra, in both domains of reversible and irreversible responses; i.e. viscoelastoplasticity. Secondly, we allow for anisotropy in both regimes; i.e. viscoelastic stiffness anisotropy, as well as orientational/directional-dependency, in the viscoplastic strengths.

Of course, the improved accuracy and material representation capabilities in these models have often been acquired at the expense of greater mathematical *complexity* and a *large* number of material parameters (introduced to describe a host of physical phenomena and complicated deformation mechanisms). In addition, the experimental characterization of these material parameters is a major factor in the successful and effective utilization of the constitutive model.

Material parameter estimation, expressed in the form of an inverse problem [8] involves the simultaneous identification of a large number of parameters from a variety of experimental tests, i.e. different loading conditions and control modes such as strain-, stress-, and mixed-controls). Such problems are known to be both mathematically and computationally challenging [7]. Adding to this difficulty is the fact that most of the material parameters lack an obvious or direct physical interpretation and they differ in scale for a given model. Also, even under load histories in simple laboratory tests, many parameters will highly interact with each other, affecting the model response predicted.

Research work in the area of model parameter fitting is rather limited [7,8,14,15,23,26,32]. In particular, specific guidelines for systematic determination of these material parameters are presently lacking. Therefore, an urgent and obvious need exists for a systematic development of a general methodology for constitutive material estimation and indeed this provides a major motivation for the work reported here.

1.2. Objectives and outline

The primary objective of this research is to develop an automated, systematic, and computationally efficient methodology to identify the material parameters for a class of complex anisotropic viscoplastic models. As a result, the integrated software named COMPARE (COnstitutive Material PARameter Estimator) has been developed. The overall strategy is outlined as follows:(i) primal analysis tools (response functionals) for the differential form of the constitutive model, (ii) sensitivity analysis, (iii) optimization technique of the associated error function, and the (iv) graphical user interface. With regard to item (i), a previously developed, complete potential-based viscoplastic model and its associated implicit integration algorithm is adopted here with the important extension to composite anisotropy [3,6,24,25]. In connection with item (ii), a direct differentiation approach is used in which the response sensitivity array is directly linked to the algorithmic tangent stiffness arising from the integrated fields in the implicit update/time stepping scheme. The optimization module is used to cast the estimation of the material parameters as a minimum-error, weighted multi-objective, non-linear optimization problem, which is subsequently solved using the sequential quadratic programming technique. The complete program has been designed to be robust, reliable, easy-to-operate, and portable.

An outline of the remainder of the paper is as follows. In Section 2, the four key elements, i.e. primal analysis, sensitivities, optimization, and graphical user interface are briefly discussed. In Section 3, the general anisotropic, hereditary multi-mechanism-based model is presented. Details of the stress update algorithm for the implicit integration of the rate equations are also given. The associated sensitivity calculations of the direct, recursive type are shown in Section 4. Numerical results and simulations are presented in Section 5.

2. The COMPARE program

The organization of the computer program COMPARE is as follows. As mentioned above, COMPARE consists of four modules; (1) primal analysis, (2) sensitivity analysis, (3) optimization and (4) user interface. A central processing routine links these four modules to formulate the estimation of the material parameters as a minimum-error (weighted multi-objective) optimization problem, which is then solved using the sequential quadratic programming technique. Each of these modules is presented in detail in the following sections.

2.1. Primal analysis

The primal analysis module in COMPARE is responsible for simulating the material response under the various conditions of an experimental test. The module takes the form of a small, non-linear finite element code utilizing a single, plane-stress element. The finite element format has been chosen to allow the flexibility of adding material models to COMPARE and to allow the relatively easy transfer of models from COMPARE to commercial finite element codes, in the form of ABAQUS UMATs for example. In addition, the finite element basis allows sufficient generality in handling different control modes (stress/strain/mixed) under multi-axial (two-dimensional, 2D) stress/strain conditions, yet utilizing a unified, strain-driven format for the implementation of the stress update algorithm. The primal analysis module has been written to be as numerically robust