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Sensitivity analysis of thin-walled members, problems and applications

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Abstract

A review of problems related to sensitivity analysis of thin walled members with open monosymmetric or bisymmetric cross-section is presented. The restraints imposed on angle of cross-section rotation, transverse displacement and cross-section warping are taken into account. The consideration is based upon the classical theory of thin-walled beams with nondeformable cross-section. The first variations of state variables due to a change of the design variable are investigated. Arbitrary displacement, internal force or reaction of the member subject to static load, critical buckling load, frequency and mode of torsional vibration are assumed to be the state variables. The dimensions of the cross-section, the material constants, the restraints stiffness, and their locations, position of the member ends are taken as the design variables. Accuracy of the approximate changes of the state variables achieved by sensitivity analysis is also discussed.

Keywords: Thin-walled members; Static analysis; Buckling; Torsional vibrations; Restraints; Sensitivity analysis

1. Introduction

The behaviour of thin-walled members is described by means of so-called state variables such as: displacements, internal forces, reactions, critical buckling loads and frequencies and modes of free vibrations. The values of these state variables depend on many parameters of the members, known as design variables *d*. In many problems of engineering practice it is very useful to know a direct relation between

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the state variable variation δs and the design variable variation of δd. Sensitivity analysis (see Haug, Choi, Komkov [1]) enables one to derive such relations. Since the sensitivity analysis of structures undergoing bending and compression or tension is well developed, the present paper deals with the sensitivity analysis of thin-walled members subjected to torsion. From the mathematical point of view, one can distinguish two kinds of design variables:

- continuous variables, for example, the cross-section dimensions and the member material constants,
- discrete variables, for instance, the restraints stiffness and their location and the support position.

In case of the variation of the continuous design variable the first order variation of the state variable sought can be expressed as follows

$$\delta s = \int_0^l F_{sd}(z)\, \delta d\, dz, \tag{1}$$

where the function $F_{sd}(z)$ can be considered as the influence line of the state variable variation due to the unit point variation of the design variable. If the discrete design variables are taken into account, then a similar relation between the state variable variation and the vector of the design variable variation $\delta\mathbf{d}$ is

$$\delta s = \mathbf{W}_{sd}^T\, \delta\mathbf{d}, \tag{2}$$

where the vector $\mathbf{W}_{sd}$ consists of the first order sensitivity coefficient corresponding to the design variable and $(...)^T$ denotes transposition of the vector.

The usual assumptions of the classical theory of thin-walled members with non-deformable cross-section (Vlasov [2])) adopted in this paper are:

1. the member cross-section is not deformed in its plane but it is subject to warping in the longitudinal direction,
2. the shear deformation in the middle surface vanishes,
3. the deformations and the strains are small,
4. the static loads are conservative,
5. the member material is homogeneous, isotropic and obeys Hooke's law.

Because of the lack of a general theory of thin-walled members with arbitrary variable cross-section, the sensitivity analysis is restricted to the member cross-section with a single or double axis of symmetry. It is well known that for the bisymmetric cross-section, torsion of the member can be considered independently of bending. In the case of monosymmetric cross-sections bending with respect of the symmetry axis and torsion are mutually dependent while bending with respect to the second axis and torsion are independent. Three types of elastic restraints are considered in this paper: the flexural restraint against the lateral displacement of the