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Sensitivity analysis as a tool for the implementation of a water quality regulation based on the Maximum Permissible Loads policy

R. Pastres[a,*], S. Ciavatta[a], G. Cossarini[b], C. Solidoro[b]

[a]Dipartimento di Chimica Fisica, University of Venice, Dorsoduro 2137, Venezia 30123, Italy

[b]Osservatorio Geofisico Sperimentale, P.O. Box 2011, 34016 Opicina, Trieste, Italy

Abstract

This paper shows how local sensitivity analysis, in respect of the parameters which specify the boundary conditions, can be used for relating the total load of non-conservative pollutants to their distributions within a water body. The method is applied to the estimation of the Maximum Permissible Load of inorganic nitrogen in the lagoon of Venice, that is of the maximum load of nitrogen which keeps its average yearly concentration below a prescribed threshold. The use of the spatial distributions of sensitivity coefficients in order to rank the sources of pollution and to forecast the effect of a reduction in the pollution is also discussed.

Keywords: Sensitivity analysis; Maximum Permissible Loads; Dissolved inorganic nitrogen; Lagoon of Venice

1. Introduction

Since 1999, in Italy the pollutant loads which enter the water-bodies have been disciplined by means of the so-called maximum-permissible-loads (MPLs) policy. Within this framework, Local Authorities should make an inventory of the sources of pollution and then fix the level of emission of each activity, in order to maintain the concentrations of potentially dangerous substances below prescribed thresholds, called 'Quality Targets' (QTs). The implementation of this policy may clearly benefit from the use of mathematical models, which can be used as tools for both estimating the MPLs, by solving the so-called' inverse problem', and exploring the consequences of different input scenarios.

In fact, in mathematical terms, the loads are specified by a set of boundary conditions: numerical models can then be used for determining a functional relationship between the set of parameters which specify the boundary conditions and the output variables which one decides to compare with the QTs. Once this task has been accomplished, one can invert this function in order to estimate the MPLs which are compatible with the targets.

These problems are investigated in this paper using the lagoon of Venice as a case-study and the sensitivity analysis in respect of each source of pollution as a tool. In fact, because of its peculiarity, this system was thoroughly investigated and a 3D reaction–diffusion water quality model is already available [1]. Advective transport is not included in the model, as numerical simulations suggested that the residual currents in the lagoon of Venice are very small [2–4]. However, diffusivities embody information about the contribution of the tide to the dispersion, as they are computed at each grid point on the basis of a statistical analysis of the results of a Lagrangian particle dispersion model [5].

The reaction–diffusion Eq. (1) is solved using a finite-difference scheme

$$\partial \mathbf{c}(x,y,z,t)/\partial t = \nabla(\mathbf{K}(x,y,z)\nabla \mathbf{c}(x,y,z,t)) + \mathbf{f}(\mathbf{c}(x,y,z),\beta,t) \quad (1)$$

In Eq. (1), **c** is the state vector; **K**, the tensor of eddy diffusivities; **f**, the reaction term; β is the set of site-specific parameters. The model simulates the dynamic of the ecosystem up to the second trophic level by using 12 state variables. The state vector includes the concentrations of the two main forms of inorganic nitrogen, ammonium and nitrate, as well as the inorganic reactive phosphorous one. These chemicals are considered to be the main cause of the eutrophication and, therefore, the current legislation fixes their QT for the lagoon of Venice. In this paper, we have applied the method outlined below to the estimation of

* Corresponding author. Tel.: +39-41-2578-674; fax: +39-41-2578-594. *E-mail address:* pastres@unive.it (R. Pastres).

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the MPLs of ammonium and nitrate: the sum of their concentration gives the dissolved inorganic nitrogen (DIN), as the concentrations of nitrite is very low. At present, the concentration of DIN is above the target, while the concentration of reactive phosphorous is very close to it, as its use in detergents was prohibited in 1989.

Ammonium, NH_4, and nitrate, NO_3, are carried into the lagoon by the rivers, and are directly released from the Industrial area of Porto Marghera, on the edge of the lagoon, and from the city of Venice and the nearby islands. The yearly evolutions of these inputs were modelled using Von Neumann-type time-dependent boundary conditions: the fluxes Φ_i were specified using a set of trigonometric polinomia

$$\Phi_i^{NH_4}(t) = \alpha_{i,0}^{NH_4} + \sum_{j=1}^{3} [\alpha_{i,2j-1}^{NH_4} \cos(2\pi jt/365) \times \alpha_{i,2j}^{NH_4} \operatorname{sen}(2\pi jt/365)] \quad (2)$$

$$\Phi_i^{NO_3}(t) = \alpha_{i,0}^{NO_3} + \sum_{j=1}^{3} [\alpha_{i,2j-1}^{NO_3} \cos(2\pi jt/365) \times \alpha_{i,2j}^{NO_3} \operatorname{sen}(2\pi jt/365)] \quad (3)$$

where $\Phi_i^{NH_4}(t)$ and $\Phi_i^{NO_3}(t)$ are the daily fluxes of ammonium of nitrate released by the ith source at time t, expressed in days.

The parameters $\alpha_{i,0}^{NH_4}, \ldots, \alpha_{i,6}^{NH_4}, \ldots, \alpha_{i,0}^{NO_3}, \ldots, \alpha_{i,6}^{NO_3}$ were estimated for each source by means of a least squares regression of the monthly data reported in Ref. [6] and on total loads data presented in Ref. [7]. The exchanges with the Adriatic sea at the three inlets were described by means of Dirichlet-type boundary conditions: the concentrations of ammonia and nitrate at the boundaries were taken from Ref. [8]. On the basis of the available data, it was possible to define 16 sources, which are shown in Fig. 1 together with the sampling stations which were monitored from 1995 to 1999. Therefore, $2 \times 7 \times 19$ parameters had to be considered as potential input factors, if one wishes to take into consideration also the fluxes through the three inlets.

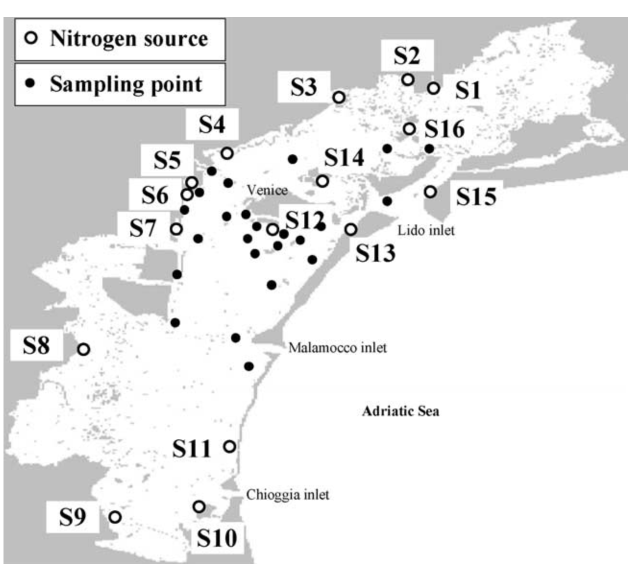

Fig. 1. Sources of DIN, white circles, and the monitoring network, filled circles.

2. Method

Global sensitivity analysis methods, based on Monte Carlo techniques [9,10], could hardly be used in this case for exploring the dependence of the state of the system on the variation of the set α of parameters which specifies the loads, because each run of the model requires about 5400 s to simulate 1 year on a 2-CPU Digital AU533MHz WS. Therefore, we decided to use local sensitivity analysis in order to

- estimate the role of each source regarding the determination of the concentration of DIN in a given portion of the water body;
- estimate the MPL for DIN, MPL^{DIN}, given the water quality target, QT^{DIN};
- obtain a first approximation of the evolution of the system within the MPL^{DIN} input scenario;
- analyse the consequences of different policies concerning the reduction of the load of nitrogen.

In order to simplify the problem, we supposed that the QT^{DIN} had to be respected for the yearly average concentration of the system: this interpretation of the law was arbitrary, but, at the moment, no precise statement exists about the type of average which has to be compared with the QT. In this case, one has to take into consideration only the first parameters $\alpha_{1,0}^{NH_4}, \ldots, \alpha_{i,0}^{NH_4}, \alpha_{ns,0}^{NH_4}, \alpha_{1,0}^{NO_3}, \ldots, \alpha_{i,0}^{NO_3}, \alpha_{ns,0}^{NO_3}$, for the ns manageable sources (ns = 16). To be concise, in the following development the second subscript index is dropped and the parameters are regrouped in a single vector, by setting $\alpha_i^{NO_3} = \alpha_{ns+i}$.

The sensitivity equation for a generic parameter α_i reads as

$$\partial \mathbf{S}_i(x,y,z,t)/\partial t = \nabla(\underline{\mathbf{K}}(x,y,z)\nabla \mathbf{S}_i(x,y,z,t)) + \underline{\mathbf{J}}\mathbf{S}_i \quad (4)$$

where $\mathbf{S}_i = \partial \mathbf{c}/\partial \alpha_i$ is the sensitivity vector, which components are the sensitivities of each state variables with respect to the parameter α_i, and $\mathbf{J} = \partial \mathbf{f}/\partial \mathbf{c}$ is the Jacobian matrix of the vector function $\mathbf{f}$. The sensitivity vector depends on the choice of the reference state around which the linearization is performed, as the reaction term of Eq. (4) is non-linear. The set of $2 \times$ ns vector Eq. (4) is solved by means of the direct method [11]. The partial derivatives which form the Jacobian matrix are calculated using symbolic calculus: this may appear to be a limit, with regard to extending this approach to other problems, but such calculations are now performed automatically by