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# Elimination of end effects in empirical mode decomposition by mirror image coupled with support vector regression

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ABSTRACT

The treatment of end effects is one of the most important open problems related to the EMD (Empirical Mode Decomposition) method. This work proposes a new approach that couples the mirror expansion with the extrapolation prediction of regression function to solve this problem. The algorithm includes two steps: the extrapolation of the signal through Support Vector (SV) regression at both endpoints to form the primary expansion signal, then the primary signal is further expanded through extrema mirror expansion and EMD is performed on the resulting signal to obtain reduced end effects. If there is not enough length for the signal to meet the need of finding the length of the data available for expanding the signal, a direct extrapolation towards the outside of the signal at the endpoint is executed by the estimate model, and the length of extrapolation points is controlled by the first local extremum. Applications of the proposed approach to the decomposition of a digital modeling signal and three segment signals from the observed earthquake signal by the EMD method are presented, and all of the results are compared with those on the basis of the traditional mirror expansion approach and the extrapolation estimate expansion based on the SV regression, which shows that the most satisfactory result can be obtained for the elimination of end effects in EMD method by mirror image coupled with SV regression.



## 1. Introduction

The method of Empirical Mode Decomposition (EMD), also called as Huang transform as was firstly introduced by Huang et al. [1], aims to analyze multicomponent signals by breaking them down into a number of elementary amplitude and frequency modulated zero mean signals referred to as intrinsic mode functions (IMFs). The elementary signals are virtually monocomponent leading to meaningful instantaneous frequency estimates through the Hilbert transform or other alternative approaches [2]. EMD results in an adaptive signal-dependent time-variant filtering procedure able to extract signal components which significantly overlap in time and frequency [3]. The physical meaning of the intrinsic processes underlying the complex signal is often preserved in the decomposed signals since the results are not prejudiced by predetermined basis and/or subband filtering process. Due to its interesting features, EMD has received much attention in recent years [4–8].

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However, the EMD technique lacks a well-established theoretical analysis which would permit a convergence proof and a direct systematic optimization of the method. Due to the nature of EMD and the obscure way it operates, the so far published modifications of the initially proposed algorithm leading to performance improvement are limited [9]. Affirmatively, some efforts have been made to the best spline implementation [2] and the detection of the optimum positions of the interpolation points [10]. In fact, the end effect is also one of the most important open problems related to the EMD method [3]. Traditional method in dealing with the end is to use window, which would mask the ends and force the ends to be tapered to zero.

For any time series signals with finite length, there are some end effects more or less in their decompositions or transforms. Although they can be reduced through the modified versions of their algorithms in which special treatments are introduced at points close to an endpoint so effectively as to avoid the propagation of end effects into the inner data [11], these treatments have some limitations in practical applications and expansion of the signal data is still the best basic approach to the solution of end effects.

This expansion mainly can be classified as three types, expansion with mirror image signals [3,8,12], expansion based on characteristics of signal waveforms [1,13] and expansion of extrapolation estimation [14–16]. Adding a mirror image of a signal at its end, referred to as expansion with mirror image signals, can expand this signal with two cases of mirror image signals created by even expansion due to the symmetry of the original signal with respect to the endpoint and by odd expansion due to the antisymmetry of the original signal with respect to the endpoint [8]. Since the sifting process is only dependent on local extrema of the signal decomposed by the EMD method, the extrema expansion is required in practice in which adding extrema is performed by mirror symmetry with respect to the extrema closest to the ends [3]. An alternative procedure based on the expansion with mirror image signals is mirror periodic expansion that can extend the signal once forever [12]. If the signal does not have an obvious symmetric waveform, its result is not as good as that from the traditional expansion procedure of mirror image. The expansion based on characteristics of signal waveforms was initially proposed by Huang et al. [1]. They added characteristics waves to the treatment of end effects, in which the extra points are determined by the average of $n$-waves (usually $n=3$) in the immediate neighborhood of the ends. However they did not give a clear description how to determine an appropriate characteristic wave and which way should be taken in this process. Recently, Wang and Liu investigated another possible way to perform the expansion based on characteristics of signal waveforms by means of the minimum similarity distance of a given signal [14]. Different from the former two methods, expansion of extrapolation estimate firstly carries out a process theoretically modeling the known signal to obtain the quantitative relationship that can describe its changes, and then calculates the data outside of the endpoint by extrapolating this relationship to realize the expansion of this signal. The related works are concerned with applications of the Artificial Neural Network (ANN) [14], Auto-Regressive and Moving-Average (ARMA) modeling [16], polynomial regression [17] and so on. In this method, reliability of the extrapolation estimate immediately determines the actual effect of expanding the signal.

Relatively, the expansion with mirror image signals has an obvious characteristic of intuition and is easier to be put into practice, and the expansion based on characteristics of signal waveforms seems to be more appropriate for the requirement to describe the complexity of problems, but it is very difficult to find out general rules that can serve as the algorithm with good performance in the treatment of end effects. The expansion of extrapolation estimate, based on time series models, has better theoretical completeness and also has a powerful potency taking account of signal waveform features, and moreover extrapolation forecasting approaches with good performance have remained a challenge waiting for further solutions. The support vector (SV) method recently proposed for estimating regressions provides a universal new tool for solving multidimensional function estimation problems such as the time series signals stochastically varying with time [17]. Many studies show that this novel type of function representation opens new opportunities for solving various problems of function approximation and estimation [18–20]. Cheng et al. [21] even applied this method to the signal analysis of EMD method and the elimination of end effects in its Hilbert spectral calculation, obtaining a quite satisfactory result. As for extrapolation through the fitting function of regression, reliability of the estimation at a given point will sharply decrease as its distance away from the known data set increases, and thus it is necessary to be careful in expanding a signal only by adding the extrapolation data to it. There is no doubt that the data at those points very close to the endpoints can hold very high credibility generally. Therefore, they should be of great applicable value in the treatment of end effects related to the EMD method.

This work proposes a new approach that couples the mirror expansion with the extrapolation prediction of regression function to the treatment of end effects. It can use advantages of these two approaches to guard result of the elimination of end effects in the EMD. The remainder of the paper is structured as follows. First, the EMD method is briefly introduced, and then in the second part the characteristics of the phenomenon of its end effects are primarily analyzed. Next, basic algorithm of the mirror expansion is introduced. Some limitations of this approach to practical applications are explained through an example of the EMD analysis of an observed earthquake signal. In the fourth part, the principle of the expansion of extrapolation estimate based on the SV regression is given in detail. The result of its application to an example of the digital modeling signal shows good performance for the extrapolation prediction. Three segments separated from the observed earthquake signal are taken as analyzed signals, and tests of their end extrapolation prediction illustrate the reliability to expand them by adding the prediction data close to the endpoints to the corresponding signals for the primary expansion. By using a signal from the primary expansion, its secondary expansion is realized through the mirror expansion, which forms the proposed approach of the mirror expansion coupled with the