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A stochastic dynamic programming approach-based yield management with substitution and uncertainty in semiconductor manufacturing

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ABSTRACT

Yield management is important and challengeable in semiconductor industry for the quality uncertainty of the final products. The total yield rate of the semiconductor manufacturing process is uncertain, each product is graded into one of several quality levels according to performance before being shipped. A product originally targeted to satisfy the demand of one product may be used to satisfy the demand of other products when it conforms to their specifications. At the same time, the products depreciate in allocation periods, which mainly results from technical progresses. This paper studies the semiconductor yield management issue of a make-to-stock system with single input, multi-products, multi-demand periods, upward substitution and periodic depreciation. The whole time horizon of the system operation process can be divided into two stages: the production stage and the allocation stage. At the first stage, the firm invests in raw materials before any actual demand is known and produces multiple types of products with random yield rates. At the second stage, products are classified into different classes by quality and allocated in numbers of periods. The production and allocation problem are modeled as a stochastic dynamic program in which the objective is to maximize the profit of the firm. We show that the PRA (parallel allocation first, then upgrade) allocation policy is the optimal allocation policy and the objective function is concave in production input. An iterative algorithm is designed to find the optimal production input and numerical experiments are used to illustrate its effectiveness.

1. Introduction

The more challengeable environment of a semiconductor industry can be characterized by randomly periodic demand, high manufacturing lead time, the expensive set-up costs, and the rapid change of technology, all of which means a significant capital and big risk. The first step in the production of semiconductor chips is the drawing of ingots of either silicon or gallium arsenide. These ingots are sliced into wafers. After several layers of semiconductor materials are placed on the wafers, they are cut into individual chips. Depending on the complexity of the circuits involved, each wafer may yield between 10 and 100,000 chips. The individual chips can then be measured against one or more dimensions of electrical performance and classified as different products. A more detailed description of the production process can be found in [1,2]. In other words, the products have random yields and are used to satisfy the demands of many products. These products have specification requirements that overlap. A product originally targeted to satisfy the demand of one product may be used

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to satisfy the demand of other products when it conforms to their specifications. The total yield rate of the semiconductor manufacturing process is probabilistic. Hence, the percentage of acceptable units and the relative proportions of products in each production lot could be different from run to run. Meanwhile, a large proportion of the product mix of a semiconductor firm has a relatively short life cycle (one or two years, typically), and a semiconductor chip may loses 60% of value within the first half year of its life cycle. All these means that yield management is important and challengeable in the semiconductor industry.

Some literatures have focused on the yield management of the make-to-stock production systems. Pasternack and Drezner [3] consider a stochastic model for two products which have a single-period inventory structure and which can be used as substitutes for each other. They prove that the expected profit function is concave, and it is possible to find optimal stocking levels for these two products. However, the deterministic yield rates assumption limits the applications of the proposed model. In practice, the yield rates are often random. Karmarkar and Lin [4], Moinzadeh and Lee [5], Lee and Yano [6], Henig and Gerchak [7] study the substitution and allocation problems with assumptions that the yield rate of one product is random and other yield rates are deterministic. Gerchak et al. [8] model a production system with random yield rates and two kinds of products, and the products can be substituted with each other. Their study focuses on identifying the properties of their model. Hsu and Bassok [9] first present a single-period, multi-product inventory model with upward substitution, and then determine the optimal production input and the allocation policy of n products for satisfying n demand classes. Ha [10–12] consider a make-to-stock production system with several product types and demand classes. The study mainly proves that the optimal stock rationing policy is closely related to the production limits, and storage level can be characterized by the corresponding customer class. Tomlin and Wang [13] assume that supplies, demands and yield rates are random, and they allow customers to choose the second-choice products if their first choice is not available. They investigate the pricing and allocation policies in a co-production system with two-class products. In all of these studies, only the single demand period is considered in the proposed models.

Replenishment is not allowed to occur within the allocation periods in several papers. Alstrup et al. [14] study a dynamic overbooking problem with substitution and two product types. The paper proposes a two-stage model: the booking stage and the allocation stage. All demands are realized at the beginning of the allocation periods and substitution is allowed in the allocation stage. Karaesmen and van Ryzin [15] give a more general model with multiple product classes. Our study is similar to their model but with a difference that the demands are realized at the beginning of each period instead of all demands being realized at the beginning of the whole allocation stage. Bitran and Dasu [16] consider a multi-product, multi-period model with demand substitution and discrete random yield. The paper gives the structure of the optimal inventory allocation policy for a two-product, two-period problem. Bitran and Leong [17] add a new service level constraint in the model and assume that the substitution decision is made before the demand is observed. Different from the discrete random yield assumption, our model assumes a continuous random yield and does not allow the backorder.

In yield management, the firm always looks for the optimal policies for allocating the inventory among customer demand classes. Thus, some literatures consider the inventory allocation problem with multi-demand periods. Van Mieghem and Rudi [18] present a newsvendor model with multiple demand periods, and their model allows the firm to replenish inventory in each demand period. Karmarkar [19], Robinson [20], Archibald et al. [21], Frank et al. [22] and Axsater [23] also use the same replenishment policy as Van Mieghem and Rudi [18]. Bassok et al. [24]prove that the substitution is beneficial in the multi-product inventory model. Shumsky and Zhang [25] examine a multi-period inventory allocation model with substitution and describe an optimal allocation policy, and then give an approximate solution for the optimal allocation quantity in each period.

A product-specific depreciation rate is usually based on the rate implicit in financial statements, and it is used in a production or allocation system as one of control variables sometimes [26–28]. In this paper, we consider a yield management problem with different application settings from the previous literatures. Our model can be seen as an extension of the single input, multi-products, single demand period model of Hsu and Bassok [9] to a new application with multi-demand periods. This paper studies the yield management issue of a make-to-stock system with single input, multi-products, multi-demand periods, upward substitution and periodic depreciation. We assume that a single input yields n different products and there are n corresponding demands. The continuous random yield rates of the products are denoted as $\eta_1, \eta_2, \ldots, \eta_n$, respectively. The allocation stage can be divided into several demand periods (T), and the firm allocates the inventory to customers after the demands within each period are observed. In each demand period, if a particular product is out of stock, the firm might upgrade the customer with a more expensive one. Each class of product has a periodic depreciation rate r_i. We consider the single input, n products, and demands, T periods make-to-stock system, as $1 \times n \times T$ system. The objective is to find the optimal production input quantity of the system. Because customers could arrive in any period, this means that the future demands for a particular product type are still unknown, the decision making on optimal allocation and the input quantity is very difficult.

The contributions of this paper are threefold. First, it extends the previous research to a multi-product, multi-demand period make-to-stock system with upward substitution and periodic depreciation rates. Second, an effective algorithm is designed to solve the proposed stochastic dynamic programming model. Third, we prove that the objective function is concave in input quantity and show that PRA allocation policy is more profitable than other propositional allocation policies.

The rest of the paper is organized as follows. Section 2 describes the basic model, which is a stochastic dynamic programming model. In Section 3, we prove that the objective function is concave in the production input quantities, give the available allocation framework and show that the PRA allocation policy is the optimal policy for allocating inventory