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A dynamic programming algorithm based on expected revenue approximation for the network revenue management problem

Kuancheng Huang [a,*], Yu-Tung Liang [b]

[a] Department of Transportation Technology and Management, National Chiao Tung University, No. 1001, Ta Hsueh Road, Hsinchu 300, Taiwan
[b] Strategy Research Unit, Office of the President, Wan Hai Lines Ltd., 5F, No. 185, Sec. 2, Tiding Boulevard, Neihu District, Taipei 114, Taiwan

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ABSTRACT

Since American Airlines successfully applied revenue management (RM) to raise its revenue, RM has become a common technique in the airline industry. Due to the current hub-and-spoke operation of the airline industry, the focus of RM research has shifted from the traditional single-leg problem to the network-type problem. The mainstream approaches, bid price and virtual nesting, are faced with some limitations such as inaccuracy due to their suboptimal nature and operation interruption caused by the required updates. This study developed an algorithm to generate a seat control policy by approximating the expected revenue function in a dynamic programming (DP) model. In order to deal with the issue of dimensionality for the DP model in a network context, this study used a suitable parameterized function and a sampling concept to achieve the approximation. In the numerical experiment, the objective function value of the developed algorithm was very close to the one achieved by the optimal control. We believe that this approach can serve as an alternative to the current mainstream approaches for the network RM problem for airlines and will provide an inspiring concept for other types of multi-resource RM problems.

1. Introduction

Revenue management (RM) has become common practice in the airline industry ever since American Airlines successfully applied several RM techniques to raise its revenue. For example, it has been estimated that RM practices generated an additional revenue of US$1.4 billion for American Airlines over a 3-year period around 1988 (Smith et al., 1992). In today's market it is very difficult for any major airline to operate profitably without RM, since according to most estimates the revenue gained by applying RM is about 4–5%, which is comparable to many airlines' total profitability in a good year (Talluri and van Ryzin, 2004). Nonetheless, how to realize the basic concept of RM, selling the right seat to the right customer at the right price, remains a challenge.

Due to the current hub-and-spoke operation of the airline industry, the focus of RM research has shifted from the traditional single-leg problem to the network problem. With multiple types of products and resources, the decision of how to sell one type of product is complicated by its impact on the future sales of the product types sharing the same resource(s). The problem complexity and the associated computational load make it impossible to derive the optimal control for a problem of practical size. The mainstream approaches, bid price and virtual nesting, have some limitations such as the inaccuracy due to their suboptimal nature and the operation interruption caused by the required updates. This study developed an algorithm

* Corresponding author. Tel.: +886 3 5731928; fax: +886 3 5720844.
E-mail address: kchuang@cc.nctu.edu.tw (K. Huang).

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to generate a seat control policy by approximating the expected revenue function in a dynamic programming (DP) model. In order to deal with the issue of dimensionality in a network context, this study used a suitable parameterized function and a sampling concept to achieve the approximation.

The remainder of this paper is organized as follows. Section 2 provides the background of the problem and reviews the related literature. The DP model for the network RM problem and the algorithm based on a parameterized function are presented in Section 3. The numerical experiment is described in Section 4. Finally, the findings of this study are summarized and conclusions are drawn in Section 5.

2. Background and prior research

Most early seat-inventory control researches relied on the following six assumptions: (1) sequential booking classes, (2) low-before-high-fare booking arrival pattern, (3) statistical independence of demands between booking classes, (4) no cancellation or no-shows, (5) single flight leg, and (6) no batch booking (McGill and Van Ryzin, 1999). For example, Belobaba (1989) developed the Expected Marginal Seat Revenue (EMSR) heuristic. The key concept of the EMSR approach is to compare the marginal value of the seat with the ticket price of the fare class when making the accept/deny decision. Although it only provides the optimal solution for the two-fare case, one advantage of the EMSR approach is that its implementation is relatively easy. In addition, the generated solution appears to be very close to the optimal solution. Nonetheless, Curry (1990), Wollmer (1992), Brumelle and McGill (1993) further developed the method in order to find the global optimal solution.

These above researches are often referred to as the static models, since the demand for each fare class is modeled by a random variable, based on the above first and second assumptions. These two assumptions greatly simplify the complexity of the RM problems, but inevitably leave some demand characteristics overlooked. In order to incorporate the time-dependent characteristic of demand, Lee and Hersh (1993) developed a DP model in which the request probability based on the Poisson arrival process is used to represent the demand pattern. Thus, the assumption of the sequential arrival for booking classes is relaxed and the booking patterns for different classes, characterized by the request probabilities indexed by booking periods, are allowed to overlap in time. Lee and Hersh (1993) further generalized the sixth assumption from single-seat booking to batch booking, and thus the request probability turns out to be dependent upon the booking size as well. Finally, Subramanian et al. (1999) developed a DP model to take into account cancellation and no-show in the fourth assumption and considered the penalty due to overbooking, a classic method to counter cancellation and no-show. The network RM problem relaxes the fifth assumption to handle a problem of multiple products with shared resources. Although the features of batch booking and cancellation/no-show are not considered in the formulated problem for this study, they can be incorporated by the techniques developed by Lee and Hersh (1993) and Subramanian et al. (1999).

The most popular approach for the network RM problem is the bid-price control (Williamson, 1992). A bid price is attached to each leg, and a booking request for a fare class of an origin–destination pair (later referred to as an ODF) is accepted if its revenue is greater than the sum of the bid prices of the used legs. The key issue for most bid-price based algorithms is finding a suitable set of bid prices, supposedly depending on the available leg seats and the time periods left before departure. Williamson (1992) set the bid prices as the dual prices of the leg capacity constraints in a linear programming (LP) model, which was basically a static model. This generally required frequent updates of bid prices during the booking process. The other issue associated with Williamson (1992) was that the stochastic feature of the demand was overlooked in the deterministic LP model. Several researches (*e.g.*, Adelman, 2007; Bertsimas and Popescu, 2003; Klein, 2007; Talluri and van Ryzin, 1999; Topaloglu, 2008; Kunnumkal and Topaloglu, 2010) have developed sophisticated algorithms to generate better bid prices for addressing the dynamic and/or stochastic feature of the demand. In addition, Talluri and van Ryzin (1998) and Topaloglu (2009) discussed the characteristics of the asymptotic optimality of the bid-price control.

The other popular approach for the network RM problem is virtual nesting, which is more suitable for being integrated with the pre-existing leg-based RM systems of many airlines. The network RM problem is decomposed into subproblems for the individual legs through mapping the multi-leg ODFs to the fare groups in the single-leg problems, which are handled by the control of booking limits or protection levels. The associated mapping (or called as indexing) process and the resulted control scheme are more complicated when compared with the simple decision rule of the bid-price control. In addition, the estimation of the seat value, which should be time- and state-dependent, remains an essential task for this type of control. Several researchers, such as Bertsimas and de Boer (2005), van Ryzin and Vulcano (2008), and Erdelyi and Topaloglu (2009), developed sophisticated virtual-nesting algorithms to handle the dynamic feature of the demand and the interrelationship among resources and products inherited in the network RM problem.

The discussion of the advantages and disadvantages of the bid-price control and the virtual nesting can be found in the chapter dedicated to the network RM problem in Talluri and van Ryzin (2004) and Phillips (2005). However, the present study does not use these two indirect control mechanisms and directly works on approximating the expected value to generate the seat-inventory control policy.

3. Mathematic models and seat control algorithms

In an airline network, an ODF can utilize the seats of multiple legs, and a seat on a leg is usually shared by multiple ODFs. The network RM problem incorporating this network feature and the dynamic characteristics of the demand can be