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Topological sensitivity analysis

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Abstract

The so-called topological derivative concept has been seen as a powerful framework to obtain the optimal topology for several engineering problems. This derivative characterizes the sensitivity of the problem when a small hole is created at each point of the domain. However, the greatest limitation of this methodology is that when a hole is created it is impossible to build a homeomorphic map between the domains in study (because they have not the same topology). Therefore, some specific mathematical framework should be developed in order to obtain the derivatives. This work proposes an alternative way to compute the topological derivative based on the shape sensitivity analysis concepts. The main feature of this methodology is that all the mathematical procedure already developed in the context of shape sensitivity analysis may be used in the calculus of the topological derivative. This idea leads to a more simple and constructive formulation than the ones found in the literature. Further, to point out the straightforward use of the proposed methodology, it is applied for solving some design problems in steady-state heat conduction.

Keywords: Topological derivative; Topological-shape sensitivity analysis; Shape sensitivity analysis; Topology optimization

1. Introduction

Many physics phenomena can be modelled by a set of partial differential equations with proper boundary conditions (boundary-value problem) or by its equivalent weak form defined over a certain domain. A question of great importance, that has awaken a lot of interest in recent years, is the ability to obtain automatically, in agreement with some measure of performance (cost function), the optimal geometry of the domain of definition of the problem under analysis. Conceptually, the problem is to find the domain, i.e. its shape and/or topology such that the cost functional is minimized subject to constraints imposed by, for example, the boundary-value problem. An already established method in the literature that addresses this kind of problems is to parameterize the domain of interest followed by an optimization with respect to these parameters. This leads to the well-known shape optimization technique. The inconvenience of this approach

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is that the topology is fixed throughout the optimization process. In order to overcome this limitation, topology optimization techniques were developed where very little is assumed about the initial morphology of the domain. This issue has received special attention over the past years since the publication of the papers by Bendsøe and Kikuchi [1] and Bendsøe [2]. The main advantage of this methodology is that the optimal topology can be obtained even from an initial configuration that is far away from the optimal one. For an overview of the area of topology optimization of continuum structures, the reader is referred to the review paper by Eschenauer and Olhoff [5], where 425 references are included.

Important contributions in the field of topology optimization have been obtained by characterizing the topology as a material density to be determined. In these methodologies the cavities correspond to a region of zero density while the domain is identified by the region where the density is non-zero. This approach is based in the concepts of relaxed formulations and homogenization techniques (see, for instance, Bendsøe and Kikuchi [1]), where, in order to obtain different densities throughout the domain, a class of microcells of laminated material is introduced and an homogenization method is used to compute the physical properties of these microstructures. Therefore, the optimal solution may be seen as a distribution of fictitious materials that compose the domain. Finally, penalization methods and filtering techniques are needed to retrieve the feasible design.

More recently, Eschenauer and Olhoff [6], Schumacher [16], Céa et al. [4], Garreau et al. [9,10] and Sokolowski and Żochowski [18,19] presented a method to obtain the optimal topology by calculating the so-called topological derivative. This derivative is a function defined in the domain of interest where, at each point, it gives the sensitivity of the cost function when a small hole is created at that point, Fig. 1. Following the paper by Eschenauer and Olhoff [5], the topological derivative concept has been used to solve topology optimization problems where no restrictions concerning the nature of the phenomena as well as the boundary conditions imposed on the holes are made. However, according to the approach adopted in the referenced works, this quite general concept can become restrictive, due to mathematical difficulties involved in the calculation of the topological derivative. In fact, the work of Garreau et al. [10] introduced several simplification hypothesis. For example, the cost function was assumed to be independent of the domain, only homogeneous Dirichlet and Neumann boundary conditions on the holes were considered, the source terms of the boundary-value problem were assumed to be constant.

On the other hand, shape sensitivity analysis, which has been shown to be a powerful tool to solve shape optimization problems, was proposed by Sokolowski and Żochowski [18] and Céa et al. [4] as an alternative way to evaluate the topological derivative. Nevertheless, their theory yields correct results only for some particular cases (for example, homogeneous Neumann boundary conditions on the hole). Moreover, in these works, the relation between both concepts was stated without mathematical proof, remaining open up to the present work.

In this work is introduced a novel definition for the topological derivative which allows to correctly use results from shape sensitivity analysis. This new approach, from now on denoted *topological-shape sensitivity analysis*, is presented in Theorem 1, which formally establishes the relation between both concepts

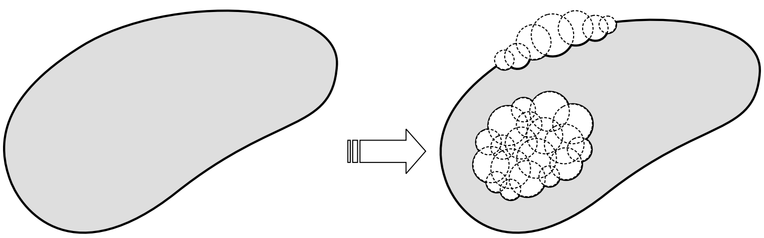

Fig. 1. Obtaining the optimal topology via topological derivative.