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Pareto optimum sensitivity analysis in multicriteria optimization

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Abstract

Multicriteria optimization problems are addressed in this paper. The gradient projection method originally used as a sort of feasible direction method is further developed and extended to the Pareto optimum sensitivity analysis. It is shown that the projected search direction defines the tangent direction of the Pareto optimum curve in the objective space. Since this method is able to identify automatically the variation of the active constraint set due to the perturbation, discontinuous derivative cases can be efficiently dealt with. To validate the method, numerical examples are solved. A comparison of the results with those obtained by the finite difference method and tangent method shows good agreement. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Multicriteria optimization is an efficient design method to deal with large-scale structural problems. To satisfy different design requirements or multidisciplinary demands, a set of conflicting objective functions are often minimized to find the compromising solution. According to the concept of Pareto optima, the utopia solution being able to minimize simultaneously each individual objective function is practically unattainable. Nowadays, the state of the art of multicriteria optimization may be reviewed in two basic aspects: developments of efficient formulations and post-processing procedures for explorations of Pareto optimum results. In general, an efficient formulation must be able to capture the entire Pareto points including nonconvex parts and ensure that they are uniformly distributed. Besides, it has to be well adapted to efficient approximation schemes and numerical optimizers. Among

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others, we can quote a variety of existing scalarization methods such as the weighing method, the compromise programming method [1], the trade-off method, the goal attainment method, the one bound min–max method as well as the multibounds method [2]. Here, attention will be focused on the Pareto optimum sensitivity analysis, which is a relatively new research subject but is of great importance. In practical applications, when the Pareto optimum solution is obtained, designers often hope to have a further understanding about the trade-off relations of different objective functions. Instead of repeating the exhaustive procedure of structural reanalyses, it is preferable to explore optimum sensitivity results for quick assessments. On the other hand, this technique seems to be very useful in multidisciplinary design optimization as Pareto optimum sensitivity permits designers to quantify properly interactions among different disciplines and to determine the influence of one objective function upon the others. For example, if the designer intends to reduce further a certain objective function at the given Pareto point, he can determine what will be the minimum sacrifices of the other objective functions. Or inversely, at the cost of a considered objective function, what will be the maximum gains of the other objective functions.

To the author's knowledge, all the previous works were mainly concentrating on the optimum sensitivity analysis with respect to problem's parameters and limited to single-objective problems. Two basic approaches exist: the optimality criteria method using the direct differentiation of the Kuhn–Tucker conditions [3] and the feasible direction method suggested by Vanderplaats and Yoshida [4]. The first one is valid only under the assumption that the set of active constraints remains unchanged, i.e., derivatives are continuous with respect to the parameter. The second one considers the parameter as an additional design variable. Compared with the first approach, the second one can give rise to proper directional derivatives if the active constraint set changes with the perturbation of the parameter. As to the multicriteria problems, the application of the gradient projection method (GPM) was first suggested by Tappeta and Renaud [5] for Pareto optimum sensitivity analysis. However, a detailed study of the work shows that the formulation and even the computing results are not so much convincing and consolidated. Additionally, the proposed computing scheme is still limited by the assumption that active constraints remain unchanged in sensitivity analysis.

The purpose of this work is to highlight involved problems in Pareto optimum sensitivity analysis, e.g., the search direction finding, the relationship between the design variable space and objective function space, the identification of the active constraint set and the computation of discontinuous derivatives. With these ideas in mind, an interactive optimum sensitivity explorer is developed using the GPM method. Numerical examples show that this post-processing procedure can be used as a complementary tool of the whole design cycle.

2. Basis of multicriteria optimization

Multicriteria optimization problems can be described by the following mathematical programming statement:

$$\text{Min } \boldsymbol{F}(\boldsymbol{X})$$

$$g_j(\boldsymbol{X}) \leqslant 0 \quad j = 1,\ m, \tag{1}$$

where $\boldsymbol{F}(\boldsymbol{X}) = [f_1(\boldsymbol{X}), f_2(\boldsymbol{X}), \ldots, f_r(\boldsymbol{X})]^{\mathrm{T}}$ is a vector-valued function composed of the set of objective functions to be minimized, $g_j(\boldsymbol{X})$ denotes the jth constraint. $\boldsymbol{X}$ is the vector of design variables.