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Discretization modeling, integer programming formulations and dynamic programming algorithms for robust traffic signal timing

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ABSTRACT

Traffic volumes are naturally variable and fluctuate from day to day. Robust optimization approaches have been utilized to address the uncertainty in traffic signal timing optimization. However, due to complicated nonlinear programming models, obtaining a global optimal solution is difficult. Instead of working with nonlinear programming models, we propose a discretization modeling approach, where the cycle, green time, and traffic volume are divided into a finite number of discrete values. The robust signal timing problem is formulated as a binary integer program. Two dynamic programming algorithms are then developed. We obtain optimal solutions for all of the instances with respect to the inputs generated from the discretization.

1. Introduction

Fixed-timed signal control assigns right-of-way at an intersection with pre-determined cycle length, numbers of phases, phase sequences, and individual phase length. While recent studies focus on real-time adaptive signal control, the fixed-timed signal control will still be used for many years due to the costs of detector installation in the adaptive signal control (Smith et al., 2002). In practice, traffic engineers often partition a whole day into several time intervals with relatively stable traffic patterns, such as morning peak, afternoon peak, etc. Certain differential equation techniques (Webster, 1958) and commercial software packages, such as TRANSYT (Vincent et al., 1980), TRANSYT-7F (Wallace et al., 1998) and Synchro (Trafficware, 2003), are then used to investigate timing plans for each time interval. A recent survey on the fixed-timed signal control as well as real-time adaptive control can be found in Papageorgiou et al. (2003).

However, real-world traffic demands are fluctuating in nature, even for the same time interval of day and day of week. Yin (2008) collected traffic data during 9–11AM for an intersection of Gainesville, Florida and showed that the traffic flow varies significantly within that time interval. Therefore, it is desirable to consider uncertain demands in the traffic signal optimization. Based on different approaches to address the issue of uncertain traffic demands, existing studies can be classified into four types.

Type I is to further partition the time interval into certain sub-intervals; and the traffic demands in each sub-interval are assumed to be constant. The traffic signal optimization is conducted over the whole analysis interval, considering the impacts of the former sub-interval on the subsequent sub-interval. The approaches by Han (1996) and Wong et al. (2002) belong to Type I.

Type II is to incorporate the mean, variance and distribution of traffic volumes into the traffic signal optimization. For example, Park et al. (2001) and Park and Kamarajugadda (2007) proposed an integration method to incorporate the mean

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and variance of traffic demands into the Highway Capacity Manual (HCM) delay equation. The integration method was then combined into a genetic algorithm to investigate timing plans.

Type III is based on different scenarios, where each scenario represents an observation of traffic demands. The occurrence probability is often assigned to each scenario. The deviation of each scenario from the mean value over all the scenarios is often included into the objective function in order to impose penalties for demand changes. Heydecker (1987), Ribeiro (1994), Ukkusuri et al. (2010), and the mean-variance model of Yin (2008) belong to Type III. The conditional value-at-risk model of Yin (2008) is also scenario-based. However, instead of penalizing the demand deviations, the conditional value-at-risk model is used against high-consequence scenarios. Zhang and Yin (2008) then extended the work to synchronize actuated signals robustly on arterial.

Type IV is related to robust optimization notions. Different from the approaches aforementioned, the robust optimization approach does not require accurate demands, demand probability distributions, or a large number of demand scenarios. Only the ranges of uncertain demands are needed. A parameter specified by users can be used to adjust the level of uncertainties. The robust optimization approach often leads to a min–max type of problem. The min–max robust optimization model of Yin (2008) belongs to Type IV.

Type I is suitable to situations where traffic volumes are relatively constant in each smaller sub-interval. Type II is appropriate if the demand probability distribution is known. When the demand distribution is unknown, Type III is more relevant if a large number of scenarios are available and the occurrence probability of each scenario is known. In Type IV, only the range of demands is needed. It is relatively easy to obtain the potential range of the uncertain parameters from historical data. For example, when the traffic volume of movement is varied, it is likely to know the potential range, say 100–200 vehicles/h. Type IV has the least requirements on the knowledge of uncertain demands and may be applicable in most situations. The robust optimization has also been applied to certain problems in the transportation sector, such as vehicle routing (Sungur et al., 2008) and road maintenance (Yin et al., 2008).

Yin (2008) completed a solid study that used a cutting plane algorithm to solve the robust signal optimization. However, as mentioned in Yin (2008), no globally optimal solution is guaranteed to be found with the cutting plane algorithm. The main difficulty is caused by the min–max model itself. Yin and Lawphongpanich (2007) prove that the cutting plan algorithm is convergent if some sufficient conditions hold. Therefore, the robust optimization problem has to be solved multiple times, each time with a different initial solution to obtain a local optimal solution.

In this study, instead of working with continuous nonlinear optimization models, we propose a discretization modeling approach and model the robust signal timing problem using binary integer programs. We show that our approach is able to obtain globally optimal solutions for the min–max model with respect to the discretized inputs.

This paper is organized as follows. Section 2 gives the discretization modeling approach. Section 3 describes the integer programming formulation and dynamic programming (DP) algorithms for the min–max model. Numerical studies are presented in Section 4. Section 5 concludes the paper and describes the future research direction.

2. The robust signal optimization and discretization modeling

Robust optimization techniques require only the potential range of the uncertain data rather than the actual distribution of the uncertain data (Ben-Tal and Nemirovski, 2002). The robust signal optimization model has an outer optimization and an inner optimization problem: the outer optimization provides potential combinations of green times and cycle length; and based on the green times and cycle length from the outer optimization, the inner optimization determines the traffic volumes in uncertain traffic volume set to maximize total traffic delays. The general structure of the robust signal optimization is as follows:

$$\begin{aligned} \min \quad & \max \; \textit{delays with respect to timings and demands} \\ & \quad \text{subject to}: \textit{uncertain traffic volume set} \\ \text{subject to}: \quad & \textit{certain constraints on green times and cycle length}. \end{aligned}$$

Defining the uncertain traffic volume set is critical to the robust signal optimization. If we conduct the optimization over the worst case scenario, the results may be over-conservative since it is rare that all of the uncertain parameters reach the worst cases simultaneously. Therefore, the robust optimization allows for limits on the potential spaces of the uncertain data. In other words, the robust optimization provide a way to balance feasibility and optimality.

We first provide some definitions. Let N be the set of lane groups. For each lane group $n \in N$, we define M_n as the set of movements. Let $|M| = \sum_{n=1}^{|N|} |M_n|$ be the total number of movements. Fig. 1 provides the signal phasing of NEMA, where movements 2, 4, 6, 8 for the through and right-turn movements, and movements 1, 3, 5 and 7 for the left-turn movements. Assume that the timing plan has four lane groups. Movements 1 and 5 are in lane group 1, movements 2 and 6 are in lane group 2, movements 3 and 7 are in lane group 3, and movements 4 and 8 are in lane group 4. Hence, $N = \{1,2,3,4\}$, $M_1 = \{1,5\}$, $M_2 = \{2,6\}$, $M_3 = \{3,7\}$ and $M_4 = \{4,8\}$. All of the movements in a lane group have the same signal timing; however, the saturation flow and traffic volumes may be different.

Let q_m^{min} and q_m^{max} be the minimum and maximum traffic volume for movement $m \in M_n$, $n \in N$, respectively. Let q_m^0 be the nominal traffic volume of lane group $m \in M_n$, $n \in N$. Yin (2008) defined the uncertain traffic volume set, Q, as