Sensitivity analysis of homogenized characteristics for some elastic composites

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Abstract

The main goal of the paper is to present theoretical aspects and the finite element method implementation of sensitivity analysis in homogenization of composite materials with linear elastic components using the effective modules approach. The sensitivity analysis of effective material properties is presented in a general form for n-component periodic composite and is illustrated using the examples of periodic 1D as well as 2D heterogeneous structures. The sensitivity coefficients are determined for the effective Young's modulus and the effective elasticity tensor components. The structural response functional for the fiber-reinforced elastic composite is proposed in the form of total strain energy resulting from some uniform strain state of the composite representative volume element (RVE). The results of sensitivity analysis presented in the paper confirm the usefulness of the homogenization method in computational analysis of composite materials and its application in composite optimization, identification, shape sensitivity studies and, after some probabilistic extensions, in stochastic analysis of random composites.

Keywords: Homogenization method; Parameter sensitivity analysis; Finite element method; Elastic periodic composites; Symbolic computations

1. Introduction

As it is known, the sensitivity analysis in engineering systems is employed to verify how input parameters of a specific engineering problem influence the state functions (displacements, stresses, temperatures, for instance) [7,8,17,19]. The sensitivity coefficients, being the purpose of such an analysis, are computed using partial derivatives of the considered state function with respect to a particular input parameter. These derivatives can be obtained starting from fundamental algebraic equations system of the problem, for instance or, alternatively, by a simple derivation if a closed form solution exists. It is important to underline that this methodology is common for all discrete numerical techniques: boundary element method (BEM), finite difference method (FDM), finite element method (FEM) as well as hybrid and meshless strategies [16].
From the computational point of view, there are the following numerical methods in structural design sensitivity analysis [5,8,17,19]: the direct differentiation method (DDM), the adjoint variable method (AVM) applied together with the material derivative approach (MDA) or the domain parametrization approach (DPA) suitable for shape sensitivity studies [4]. Considering these capabilities and, on the other hand, a very complex structure of composite materials, sensitivity analysis should be applied especially in design studies for such structures. Instead of a single (or two) parameters characterizing elastic response of homogeneous structure, the total number of design parameters is obtained as a product of components number in a composite and the number of material and geometrical parameters for a single component. Some extra state variables should be analyzed to define interfacial behavior, general interaction of the constituents and/or the lack of periodicity. Usually, to reduce the complexity of original composite, so-called effective homogenization medium having the same strain (or complementary) energy is analyzed.

This paper is devoted to general computational sensitivity studies of the homogenization method for some periodic composite materials with linear elastic and transversely isotropic constituents. The composite is first homogenized—the effective material tensor components are computed using the FEM-based additional computer program; material parameters of the composite most decisive for its effective material properties are determined numerically. It should be underlined that the homogenization method is generally an intermediate numerical tool applied to exclude the necessity of composite micro-scale discretization and, in the same time, to reduce the total number of degrees of freedom of the entire model. On the other hand, quite various numerical homogenization techniques are observed. They can be divided generally into two essentially different approaches: stress averaging (the boundary stresses are introduced between the composite constituents plus displacement-type periodicity conditions) [11–15] and strain approach (uniform extensions of the RVE boundaries in various directions plus periodicity conditions on the remaining cell edges) [6]. Considering this, different results of the homogenization method in terms of the effective material tensors are obtained and hence quite different sensitivity gradients must be computed in these two approaches. The sensitivity analysis introduces a new aspect of the homogenization technique—it can be verified if the homogenized and original structures have the same or even analogous (in terms of their signs) sensitivity gradients. Then the composites can be optimized by manipulation with its material parameters or by selection of various constituent materials with computationally determined shape to the new designed composite structure.

The sensitivity gradients are computed here by the use of homogenization-oriented computer program MCCEFF [12–15] according to the DDM approach implementation and presented as functions of the composite design parameters—Young’s moduli and Poisson’s ratios of the constituents. Since a finite difference scheme is used for the sensitivity gradients computations, numerical sensitivity of final results to the increment of arbitrarily introduced parameter must be verified. This numerical phenomenon makes it necessary to determine the most suitable interval of parameters increments for the particular effective elasticity tensor components. The entire computational methodology is illustrated by two examples—1D and 2D two component periodic composites. The closed form effective Young’s modulus is used in the first example, while the homogenization function is to be computed in the second case. Both illustrations show that different components of the effective elasticity tensor show different sensitivities to particular mechanical properties of the original composite and, further, the illustrations make it possible to determine the most decisive elastic parameters for the homogenization-based computational design studies.

Finally, it should be noted that sensitivity analysis can be used for validation of various homogenization methods. In most cases an increase of Young’s moduli of composite components should result in a corresponding increase of the effective material tensor components; the reversed phenomenon can be observed for some specific cases, but usually in an extremely small range only. Therefore, if the sensitivity analysis shows that most of the gradients are negative, the homogenization theory should be essentially corrected.

An applied effective modulus method is verified below using the examples of unidirectionally distributed heterogeneities in the periodic two-component bar structure and of a fiber-reinforced periodic composite.
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