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Design sensitivity analysis and topology optimization of displacement–loaded non-linear structures

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Abstract

A continuum-based design sensitivity analysis (DSA) method for geometrically nonlinear systems with nonhomogeneous boundary conditions is developed to topologically optimize the displacement–loaded nonlinear structures. In the adjoint variable method, the solution space requires just homogeneous boundary conditions even if the original system has nonhomogeneous ones. A design sensitivity expression for the instantaneous rigidity functional is derived for the displacement–loaded nonlinear topology optimization. The tangent stiffness is obtained at the end of the equilibrium iterations in the nonlinear analysis of the original system; this stiffness is used in the DSA so that no iteration would be necessary to evaluate the design sensitivity expressions. In force–loaded systems, the solution dose not converge easily because the material distributes sparsely sometimes during optimization. However, when the displacement–loaded system is used, there is no convergence difficulty.

Keywords: Design sensitivity analysis; Adjoint variable method; Topology optimization; Prescribed displacement; Geometrically nonlinear structures

1. Introduction

Over the past few years, many researchers have studied design sensitivity analysis (DSA) methods for structural systems. Design sensitivity is defined as the variation of performance measures with respect to the design variables [8]. In the continuum DSA approach, the design sensitivity expressions are obtained by taking the first-order variation of the continuum variational equation, which represents the structural system. The continuum DSA methods developed so far can handle several types of design variables. In this paper, a continuum DSA method for material property variation is considered for use in the topology design optimization.

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Topology optimization is a method that helps designers to find a suitable structural layout for the required structural performances. Ever since Bendsøe and Kikuchi [2] introduced the homogenization method, other methods have been developed for topology optimization. Design variables are the parameters of material distribution for each sub-domain of the discretized structural system. Therefore, many design parameters are used to find the best material distribution. The gradients of design parameters, known as design sensitivities, are required in gradient-based optimization methods.

The conventional topology optimization method finds the best design of a linear structure that yields the stiffest structure by minimizing the compliance. The structure may deform excessively because the material in the structure becomes too sparsely distributed during optimization. In linear analysis, the abnormal deformation is not a critical matter because the consequence of the analysis, i.e. deformation, is never further used. On the other hand, in nonlinear incremental analysis, the results from the previous load step are again used to proceed to the next load step. Therefore, if the previous load step yields an unrealistic deformation, the iterative solution method may have difficulty in convergence [3,9,10]. However, in this paper, prescribed displacements are used so that no such an erroneous deformation is occurred. Thus, the nonshape continuum DSA method that has homogeneous displacement boundary condition is extended to problems with nonhomogeneous displacement boundary condition. The adjoint variable method (AVM) is used to efficiently compute the design sensitivity. Displacement and rigidity are selected as the performance measures for use in the topology optimization. The developed DSA method combined with a gradient-based optimization algorithm is used in the topology optimization problems [4,5].

2. Nonlinear problems with nonhomogeneous boundary conditions

The equilibrium of a deformable body can be expressed, by using the principle of virtual work in total Lagrangian formulation [1,6,7], as

$$\int_{{}^0\Omega} S_{ij}(\mathbf{z})\bar{\varepsilon}_{ij}(\mathbf{z})\,\mathrm{d}^0\Omega = \int_{{}^0\Omega} b_i\bar{z}_i\,\mathrm{d}^0\Omega + \int_{{}^0\Gamma_\mathrm{t}} T_i\bar{z}_i\,\mathrm{d}^0\Gamma, \text{ for } \forall\bar{\mathbf{z}} \in \overline{Z}, \tag{1}$$

where $\mathbf{z}$, $\bar{\mathbf{z}}$, Z and $\overline{Z}$ are displacement, virtual displacement, trial solution space, and variational space, respectively. $S_{ij}(\mathbf{z})$, $\varepsilon_{ij}(\mathbf{z})$, b_i, and T_i are the second Piola–Kirchhoff stress tensor, the Green–Lagrange strain tensor, body force intensity, and surface traction, respectively. ${}^0\Omega$ and ${}^0\Gamma_\mathrm{t}$ are the structural domain and traction boundary at initial configuration, respectively. The spaces Z and $\overline{Z}$ are defined by

$$Z = \{\mathbf{z} \in H^1(\Omega) : z_i = \mathrm{d}_i \text{ on } \Gamma_\mathrm{d}, i = 1,2,3\} \tag{2}$$

and

$$\overline{Z} = \{\bar{\mathbf{z}} \in H^1(\Omega) : \bar{z}_i = 0 \text{ on } \Gamma_\mathrm{d}, i = 1,2,3\}. \tag{3}$$

The Green–Lagrange and virtual strain tensors are defined by

$$\varepsilon_{ij}(\mathbf{z}) = \frac{1}{2}\left(\frac{\partial z_i}{\partial^0 x_j} + \frac{\partial z_j}{\partial^0 x_i} + \frac{\partial z_m}{\partial^0 x_i}\frac{\partial z_m}{\partial^0 x_j}\right) \tag{4}$$

and

$$\bar{\varepsilon}_{ij}(\mathbf{z}) = \frac{1}{2}\left(\frac{\partial \bar{z}_i}{\partial^0 x_j} + \frac{\partial \bar{z}_j}{\partial^0 x_i} + \frac{\partial \bar{z}_m}{\partial^0 x_i}\frac{\partial z_m}{\partial^0 x_j} + \frac{\partial z_m}{\partial^0 x_i}\frac{\partial \bar{z}_m}{\partial^0 x_j}\right) \equiv \widehat{\varepsilon}_{ij}(\mathbf{z};\bar{\mathbf{z}}). \tag{5}$$

Note that the differentiation is taken in the initial configuration (0). The constitutive relation is defined by

$$S_{ij}(\mathbf{z}) = c_{ijkl}\varepsilon_{kl}(\mathbf{z}), \tag{6}$$