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Engineering Analysis with Boundary Elements 27 (2003) 671–681

ENGINEERING ANALYSIS with BOUNDARY ELEMENTS

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# Formulation of a numerical process for acoustic impedance sensitivity analysis based on the indirect boundary element method

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Received 29 October 2001; revised 22 May 2002; accepted 24 May 2002

**Abstract**

The objective of the work presented in this paper is the formulation, implementation and validation of an algorithm for computing the acoustic sensitivity with respect to the unequal impedance boundary conditions in an indirect boundary element method (IBEM). The IBEM integral equations are considered for all possible acoustic boundary conditions including velocity, pressure, unequal impedance, and simultaneous velocity and unequal impedance boundary conditions. The numerical system of equations is developed using a variational approach. The sensitivity formulation is based on analytically differentiating the system of equations formed by the variational approach with respect to the unequal acoustic impedance boundary conditions. Numerical sensitivity results obtained using the formulation developed in this paper are compared to analytical solutions in order to validate the new formulation.


*Keywords:* Indirect boundary element method; Sensitivity analysis; Unequal acoustic impedance

## 1. Introduction

The acoustic sensitivity analysis and design optimization aiming to reduce the structure-borne noise levels has been an active research area in recent years. The finite element method (FEM) [1,2] and the boundary element method (BEM) [3–5] are two major techniques for numerical acoustic analysis. Several developments have been presented in the past for computing the sensitivity of the acoustic response with respect to certain design variables [6–13]. The FEM was utilized to analyze the continuum Design Sensitivity Analysis problem for a structural acoustic system with respect to the changes in design variables such as the thickness and cross-sectional area of structural components [6]. The sensitivity of the radiated acoustic power with respect to the change of acoustic velocities applied on the surface of a given structure was studied numerically using BEM [7,8]. The design sensitivities of the radiated noise were computed using a finite difference method, where the design variables are perturbed slightly and the radiated noise is obtained using FEM [9]. An algorithm for deriving the sensitivities of the acoustic response with respect to acoustic velocity and impedance parameters was developed based on the direct boundary element method [10]. FEM and BEM were combined to analyze the structural/acoustic sensitivity with respect to structural design variables for both deterministic [11] and stochastic [12] excitations. A BEM formulation for the sensitivity analysis and shape identification of structures immersed in a fluid was also presented [13].

The sensitivity analysis of the acoustic pressure with respect to the unequal values of the acoustic impedance boundary conditions applied on either side of a thin structure constitutes a new development for the acoustic sensitivity analysis. Values for the specific acoustic impedance $z_1$ and $z_2$ are assigned to each side of the surface of the structure, respectively. In engineering applications, these values represent the acoustic absorptive properties associated with each side of the surface and typically they are different from each other. The relationship between the change of the values in $z_1$ and/or $z_2$ and the acoustic pressure at certain field points of interest can be utilized in order to apply effective acoustic treatment.

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doi:10.1016/S0955-7997(03)00026-2

The formulation of the numerical acoustic impedance sensitivity analysis presented in this paper is based on the indirect boundary element method (IBEM) [14–19]. The IBEM is well suited for computing acoustic sensitivity with respect to unequal acoustic impedance boundary conditions due to its unique capability for simultaneously modeling the acoustic medium on both sides of a thin structure. The indirect boundary integral equations are first formulated for all possible acoustic boundary conditions including velocity boundary conditions, acoustic pressure boundary conditions, unequal acoustic impedance boundary conditions, and simultaneously applied structural velocity and unequal acoustic impedance boundary conditions. The integral equations are solved using a variational approach [20]. A system of equations that relates the acoustic boundary conditions to the unknown primary variables on the surface of the boundary element model is obtained. An implicit differentiation is performed on the primary system of equations for deriving the sensitivity of the primary variables with respect to the unequal acoustic impedance boundary conditions. Thus, the differentiation is performed analytically on the functional of the integral equations formulated by the variational approach. Finally, the sensitivity of the acoustic pressure at certain field points with respect to the unequal impedance boundary conditions is computed by differentiating the boundary integral equations that relate the acoustic pressure at field points to the primary variables on the surface of the IBEM model. The sensitivity of the surface primary variables is also employed in this computation.

The acoustic impedance sensitivity capability developed in this paper is validated using an analytical solution. The acoustic plane waves propagating inside a rectangular duct are analyzed and the numerical results are compared to the readily available analytical solutions. Three different sets of boundary conditions are analyzed for a wide range of acoustic impedance values in order to validate the numerical formulation.

## 2. Theory

The integral equations formulated in IBEM for all possible acoustic boundary conditions are outlined first. A variational approach is employed for solving the integral equations numerically and for evaluating the primary acoustic variables on the surface of the boundary element model. Finally, the new numerical process for the acoustic sensitivity analysis with respect to unequal impedance boundary conditions is presented.

### 2.1. Basic integral equations

In IBEM, the acoustic primary surface variables of the formulation are defined as the difference in the acoustic pressure and the difference in the normal gradient of the acoustic pressure between the two sides of the boundary element model. The acoustic pressure at any given field point can be evaluated as a surface integral [15]:

$$p(\vec{r}) = \int_{S_Y} \left( G(\vec{r}, \vec{r}_Y)\delta\mathrm{d}p(\vec{r}_Y) - \frac{\partial G(\vec{r}, \vec{r}_Y)}{\partial \hat{n}_Y}\delta p(\vec{r}_Y) \right) \mathrm{d}S_Y \qquad (1)$$

$S_Y$ represents the surface of the boundary element model of the structure, subscript $Y$ indicates a source point on the boundary element surface, $\vec{r}$ is the position vector for the field point, $p(\vec{r})$ is the acoustic pressure at a field point defined by position vector $\vec{r}$, $\vec{r}_Y$, is the position vector of a source point on the surface of the model, $\hat{n}_Y$ is the unit normal at the location of the source point, the three dimensional free space Green's function $G(\vec{r}, \vec{r}_Y)$ is defined as:

$$G(\vec{r}, \vec{r}_Y) = \frac{1}{4\pi|\vec{r} - \vec{r}_Y|} \mathrm{e}^{-\mathrm{j}k|\vec{r} - \vec{r}_Y|},$$

$\delta\mathrm{d}p(\vec{r}_Y)$ and $\delta p(\vec{r}_Y)$ are the primary variables of the IBEM defined as the difference in the normal gradient of the acoustic pressures and the difference in the acoustic pressures between the two sides of the boundary element model, respectively, and $k$ is the wave number of the acoustic medium.

The difference in the acoustic pressures across the surface of the boundary element model is defined as [14]:

$$\delta p(\vec{r}_Y) = p(\vec{r}_{Y_1}) - p(\vec{r}_{Y_2}) \qquad (2)$$

Subscripts '1' and '2' are associated with the two sides of the boundary element surface, $p(\vec{r}_{Y_1})$ and $p(\vec{r}_{Y_2})$ are the acoustic pressures on side '1' and on side '2' of the surface at location $\vec{r}_Y$, respectively. The difference in the normal gradient of the pressures is defined as [14]:

$$\delta\mathrm{d}p(\vec{r}_Y) = \frac{\partial p(\vec{r}_{Y_1})}{\partial \hat{n}_Y} - \frac{\partial p(\vec{r}_{Y_2})}{\partial \hat{n}_Y} = -\mathrm{j}\rho\omega(v(\vec{r}_{Y_1}) + v(\vec{r}_{Y_2})) \qquad (3)$$

$\rho$ is the density of the acoustic medium, $\omega$ is the radial frequency of the analysis, $v(\vec{r}_{Y_1})$ and $v(\vec{r}_{Y_2})$ are the acoustic velocities on side '1' and on side '2' of the surface at location $\vec{r}_Y$, respectively.

The definition of the primary variables in IBEM contains information for the acoustic medium on both sides of the surface of the model. Thus, the IBEM can model a thin structure that radiates from both sides and it can also account easily for openings and multiple connections [18] in the acoustic boundary element model.

### 2.2. Acoustic boundary conditions

The integral equations in IBEM are formulated by combining the acoustic boundary conditions with the basic integral equation presented in Section 2.1. Four types of acoustic boundary conditions are considered in this paper and defined as following: