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Sensitivity Analysis of an Optimal Control Problem in Greenhouse Climate Management

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Optimal control systems are based on a performance measure to be optimised and a model description of the dynamic process to be controlled. When on-line implementation is considered, the performance of optimally controlled processes will depend on the accuracy of the model description used. Sensitivity analysis offers insight into the impact of uncertainty in the model parameters on the performance of the optimally controlled process. Additionally, sensitivity analysis may reveal the mechanisms underlying optimal process operation. This paper describes the methodology and results of a sensitivity analysis of an optimal control problem in greenhouse climate management. The methodology used, is based on variational arguments and requires a single solution of the optimal control problem, resulting in a computationally efficient technique. The example considered deals with economic optimal greenhouse climate management during the cultivation of a lettuce crop. The sensitivity analysis produced valuable insight into the performance sensitivity and operation of the controlled process. Both the model description of crop growth and production as well as the outside climate conditions have a strong impact on the performance. Humidity control plays a dominant role in economic optimal greenhouse climate management, emphasising the need for an accurate description of humidity effects on crop growth and production, either in terms of quantitative models or time-varying constraints on the humidity level in the greenhouse. Finally, the study revealed that the dynamic response times in the greenhouse climate are not limiting factors for economic optimal greenhouse climate control.

1. Introduction

The optimal control methodology is a powerful technique to facilitate the design and analysis of optimally controlled systems. Optimal control systems are based on a model description of the dynamic process to be controlled and are designed in such a way that a performance criterion is optimised with respect to the control action applied to the system (*e.g.* Pontryagin *et al.*, 1962). In practice, the structure as well as the parameter values of the model rarely coincide exactly with the real process. Since the control system is designed to be optimal with particular regard to the nominal structure and parameter values of the model used, it can be expected that the control system is sensitive to modelling errors which may reduce the performance of an optimal control system in practice. Therefore, sensitivity considerations are among the fundamental aspects of the synthesis and analysis of optimal control systems.

One way to assess performance sensitivity is to substitute one by one the original values of the model parameters by slightly perturbed values and to compute the new optimal control and corresponding value of the performance criterion. This, however, is a rather time consuming procedure. In this research, a first-order approach to the sensitivity analysis of open-loop optimal control problems was used as derived by Courtin and Rootenberg (1971) and Evers (1979, 1980). Using variational arguments, the methodology requires a single calculation of the open-loop optimal control and corresponding state and costate trajectories. These are then used to calculate a first-order approximation of the performance sensitivity, thus saving a considerable amount of computation time.

Notation

c	model parameter
$c_{ai,ou}$	heat transmission coefficient through the greenhouse cover (6·1), $\mathrm{W\,m^{-2}\,°C^{-1}}$
$c_{cap,c}$	volumetric capacity of greenhouse air for carbon dioxide (4·1), m
$c_{cap,h}$	volumetric capacity of greenhouse air for humidity (4·1), m
$c_{cap,q}$	heat capacity of greenhouse air (30000), $\mathrm{J\,m^{-2}\,°C^{-1}}$
$c_{cap,q,v}$	heat capacity per volume unit of greenhouse air (1290), $\mathrm{J\,m^{-3}°C^{-1}}$
c_{co_2}	costs of carbon dioxide (42×10^{-2}), $\mathrm{Hfl\,kg^{-1}}$
$c_{co_2,1}$	temperature effect on CO_2 diffusion in leaves ($5{\cdot}11 \times 10^{-6}$), $\mathrm{m\,s^{-1}\,°C^{-2}}$
$c_{co_2,2}$	temperature effect on CO_2 diffusion in leaves ($2{\cdot}30 \times 10^{-4}$), $\mathrm{m\,s^{-1}\,°C^{-1}}$
$c_{co_2,3}$	temperature effect on CO_2 diffusion in leaves ($6{\cdot}29 \times 10^{-4}$), $\mathrm{m\,s^{-1}}$
c_{leak}	leakage air exchange through greenhouse cover ($0{\cdot}75 \times 10^{-4}$), $\mathrm{m\,s^{-1}}$
$c_{pl,d}$	effective canopy surface (53), $\mathrm{m^2\,kg^{-1}}$
$c_{pri,1}$	parameter defining price of lettuce (1·8), $\mathrm{Hfl\,m^{-2}}$
$c_{pri,2}$	parameter defining price of lettuce (16), $\mathrm{Hfl\,kg^{-1}}$
c_q	price of heating energy ($6{\cdot}35 \times 10^{-9}$), $\mathrm{Hfl\,J^{-1}}$
c_R	gas constant (8314), $\mathrm{J\,K^{-1}\,kmol^{-1}}$
$c_{rad,phot}$	light use efficiency ($3{\cdot}55 \times 10^{-9}$), $\mathrm{kg\,J^{-1}}$
$c_{rad,q}$	heat load coefficient due to solar radiation (0.2)
$c_{resp,d}$	respiration rate in terms of respired dry matter ($2{\cdot}65 \times 10^{-7}$), $\mathrm{s^{-1}}$
$c_{resp,c}$	respiration rate in terms of produced carbon dioxide ($4{\cdot}87 \times 10^{-7}$), $\mathrm{s^{-1}}$
$c_{T,abs}$	temperature in K at 0°C (273·15), K
$c_{v,pl,ai}$	canopy transpiration mass transfer coefficient ($3{\cdot}6 \times 10^{-3}$), $\mathrm{m\,s^{-1}}$
$c_{v,1}$	parameter defining saturation water vapour pressure (9348), $\mathrm{J\,m^{-3}}$
$c_{v,2}$	parameter defining saturation water vapour pressure (17·4)
$c_{v,3}$	parameter defining saturation water vapour pressure (239), °C
$c_{v,4}$	parameter defining saturation water vapour pressure (10998), $\mathrm{J\,m^{-3}}$
c_{V_c}	perturbation parameter on carbon dioxide concentration outside greenhouse (1)
$c_{V_{rad}}$	perturbation parameter on solar radiation outside greenhouse (1)
c_{V_h}	perturbation parameter on humidity outside greenhouse (1)
c_{V_T}	perturbation parameter on temperature outside greenhouse (1)
$c_{\alpha\beta}$	yield factor (0·544)
c_σ	weighting factor in penalty function, $\mathrm{Hfl\,m^{-2}\,s^{-1}}$
c_Γ	carbon dioxide compensation point ($5{\cdot}2 \times 10^{-5}$), $\mathrm{kg\,m^{-3}}$
H	Hamiltonian, $\mathrm{Hfl\,m^{-2}\,s^{-1}}$
J	performance measure, $\mathrm{Hfl\,m^{-2}}$
m	number of model parameters
n	number of state variables
i, j, k	iteration numbers
p	penalty
p_T	penalty for constraint violations by greenhouse air temperature, $\mathrm{Hfl\,m^{-2}\,s^{-1}}$
p_c	penalty for constraint violations by carbon dioxide concentration, $\mathrm{Hfl\ m^{-2}\,s^{-1}}$
p_h	penalty for constraint violations by humidity, $\mathrm{Hfl\,m^{-2}\,s^{-1}}$
$Q_{vent,q}$	energy exchange by ventilation and transmission through the cover, $\mathrm{W\,m^{-2}}$
$Q_{rad,q}$	heat load by solar radiation, $\mathrm{W\,m^{-2}}$
R_{Xh}	relative humidity
t	time
t_b	start time of optimisation interval
t_f	end time of optimisation interval
u	control input
U_c	supply rate of carbon dioxide, $\mathrm{kg\,m^{-2}\,s^{-1}}$
U_q	energy supply by the heating system, $\mathrm{W\,m^{-2}}$
U_v	ventilation rate, $\mathrm{m\,s^{-1}}$
V_c	carbon dioxide concentration outside the greenhouse, $\mathrm{kg\,m^{-3}}$
V_h	outdoor humidity concentration, $\mathrm{kg\,m^{-3}}$
V_{rad}	solar radiation outside the greenhouse, $\mathrm{W\,m^{-2}}$
V_T	outdoor temperature, °C
x	state variable
X_c	carbon dioxide concentration in greenhouse, $\mathrm{kg\,m^{-3}}$
X_d	crop dry weight, $\mathrm{kg\,m^{-2}}$
X_h	humidity concentration in greenhouse, $\mathrm{kg\,m^{-3}}$
X_T	air temperature in the greenhouse, °C
λ	costate
λ_d	costate of crop dry weight, $\mathrm{Hfl\,kg^{-1}}$
λ_c	costate of carbon dioxide concentration, $\mathrm{Hfl\,m\,kg^{-1}}$
λ_h	costate of humidity concentration, $\mathrm{Hfl\,m\,kg^{-1}}$
λ_T	costate of air temperature, $\mathrm{Hfl\,m^{-2}\,°C^{-1}}$
$\varphi_{phot,c}$	gross canopy photosynthesis rate, $\mathrm{kg\,m^{-2}\,s^{-1}}$
$\varphi_{vent,c}$	mass exchange of carbon dioxide through the vents, $\mathrm{kg\,m^{-2}\,s^{-1}}$