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About finite element sensitivity analysis of elastoplastic systems at large strains

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Abstract

Influence of a discontinuous nature of the elastoplastic systems response at large strain onto their sensitivity with respect to a design parameter is considered in the paper. It is discussed in the framework of the finite element modelling using the direct differentiation method for the sensitivity response calculation. Elastoplastic behaviour is formulated on the additive approach of the rate of deformation tensor and a hypoelastic characterization of the elastic response. It is shown that the computed sensitivity response of the system's FE models can experience steep jumps on its overall path. This fact is confirmed by some examples.

Keywords: Sensitivity analysis; Large strain; Elastoplasticity; Finite element method; Direct differentiation method

1. Introduction

In general, a sensitivity analysis is a topic mainly used in optimisation computations, reliability analyses, inverse identification studies and process design investigations. In gradient-based algorithms that are used to solve the mentioned tasks it is actually indispensable. The sensitivity analysis results reflect always the behaviour of a physical system the analysis is concerned with. In this context, when considering specifically metal forming processes, two major system properties determine the system mechanical response, and consequently, the corresponding sensitivity analysis as well, namely large strains and path dependence. In the literature the sensitivities of such systems, referring directly to metal forming processes, have been considered in Refs. [4–8].

Because of extreme mathematical complexity of numerical models that are used in computer simulations of elastoplastic large strain systems responses, at first, the sensitivities, i.e. the directional derivatives of the problem dependent quantities with respect to a specific design variable, have been computed only by the finite difference method (FDM). As it is well known, the FDM is very simple for implementation but it is prohibitively expensive for practical use. Not only, that the FDM requires at least an additional evaluation of a system response, i.e. primal analysis, for a finite perturbation size of a particular design variable, but it usually requires also additional computations to find out the appropriate perturbation size to avoid undesirable errors associated with numerical differentiation and truncation. Adopting improperly sized variable perturbation may result in too large error at the computation of the sensitivity, and consequently also in poor effectiveness of methods used in the solution of optimisation, inverse or identification problems. Nevertheless, the simplicity of its implementation is such an advantage that the FDM is very frequently used, even nowadays, in solution of the above-mentioned problems, despite of its huge computer time consumption.

Unlike to the FDM, analytical methods such as the direct differentiation method (DDM), or the adjoint variable method (AVM), are not prone to the

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above-mentioned drawbacks. Their main advantage is that they require only a little part of the computer time needed for the primal analysis of the system. On the other hand, their formulation, being dependent also upon the kind of the primal problem and its solution scheme, is far away from being a simple task, and actually, it requires a lot of mathematical derivations.

Concerning a sensitivities' computation of elastoplastic systems, there are some issues regarding manifestation of eventual discontinuity of the sensitivities, which are worth attracting our attention. Their origin is always the duality of the possible material response, i.e. elastic or elasto-plastic, which is associated with the actual change of the stress–strain state in a material point due to a variation in the design. Namely, a material transition from the elastic to elastoplastic behaviour or vice versa is usually characterized by large difference of the response material moduli. In consequence, with the identified discontinuity at a material point the question arises whether the overall system sensitivities are also characterized by discontinuous jumps. In continuous formulations non-uniqueness of the sensitivities is certainly present at special problems (see [1,3,6]), but it is unlikely to be clearly visualized within numerical models where a numerical integration of the constitutive equations using finite time increments is considered. Actually, due to the space and time discretization the corresponding sensitivity response of the numerical models on its overall deformation path will not be smooth by definition, but unfortunately, it may experience steep jumps also when this is not consistent with the natural sensitivity response.

In the present contribution the sensitivities computation will be discussed in greater detail within the framework of the so-called standard elastoplastic formulation, which is based on the additive approach of the rate of deformation tensor and assumed hypoelastic material model. Thus, the sensitivity analysis is carried out here on a different theoretical basis as it is presented in the cited references below [4,5,7]. For example, Fourment et al. [4], have taken a more easily solvable rigid viscoplastic material model into account, while in the paper by Gelin and Ghouati [5] a comprehensive elasto-viscoplastic constitutive model, based on the multiplicative approach of the deformation gradient, has been applied, but only for the constitutive sensitivity analysis. A similar hyperelastic–viscoplastic material model, based on the multiplicative approach, has been considered also in the paper by Zabaras et al. [7], both for the shape and constitutive sensitivities. In our contribution the both types of the sensitivities will be considered as well.

In Section 2 first a mathematical derivation of the shape and constitutive sensitivity formulation using the DDM is briefly presented. The indicated problem with the arising issues is addressed in the third section, while in the last section this is numerically demonstrated by using the DDM and the FDM on a number of examples concerning the sensitivity analysis of elastoplastic systems, including also a contact interaction with rigid bodies.

2. Solution algorithm of the sensitivity problem

2.1. Introductory remarks

The essence of a sensitivity problem solution by the DDM is that the differentiation is applied directly on the numerical model, which is used for the solution of the primal problem, i.e. evaluation of the basic response variables of the elastoplastic system concerned. In view of a discrete approximation approach to the solution of the primal problem a corresponding numerical model, based on the finite element methodology, and a corresponding incremental solution procedure, enabling a step by step numerical integration of the rate form of the path-dependent problem over a given time interval, are adopted. Basically, the procedure is divided into two separate tasks, which both together solve iteratively the non-linear incremental governing equations. The first one is concerned with the determination of the actual equilibrium configuration of the system which is subjected to the action of external forces and/or driving displacements arising in a given time increment, while the second one is concerned with the determination of the stress state associated with the computed system deformation. Thus, on the global level the numerical procedure solves a variational problem, i.e. the equilibrium equations in the integral virtual work form, while at the integration points of a reference space of the system it solves a constitutive problem, which is specified by a numerically integrated system of the constitutive rate relations between the stress and deformation rates. In this sense the sensitivity problem is solved as well.

Before going on we shall explain in this subsection some facts, which clarify the idea of the sensitivity-problem solution. We shall start with the primal problem, its basic response variable being defined as

$$\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}_0, t) = \mathbf{X}_0 + \mathbf{u}(\mathbf{X}_0, t), \tag{1}$$

where the function $\boldsymbol{\chi}(\mathbf{X}_0, t)$ connects, in the Lagrange sense, the current location $\mathbf{x}$ of a material point at a given time t with its location $\mathbf{X}_0$ at time $t = 0$, when the considered system is in its unstressed, i.e. its initial configuration $\aleph_0$. The corresponding displacements undergone by the system are given by a vector field $\mathbf{u}(\mathbf{X}_0, t)$, which is interpreted, in accordance with the finite element methodology and numerical time integration used, by a series of sets with discrete nodal values at discrete time parameters t_n, $n = 1, 2, \ldots, N_{\mathrm{END}}$. The path of the