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Sensitivity analysis applied to a dynamic railroad model

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Abstract

An analytical method of analyzing sensitivity is presented. It is shown that in a special case, when the dynamical problem is described by differential equations (of any order) with constant coefficients, first and second order semilogarithmic (semirelative) sensitivity functions can be determined analytically. The method is applied to the practical problem of railway track vibration, with the intention of using it for the identification of railway track model parameters in the future. The railway track model is an infinite beam resting on multi-parameter viscoelastic subsoil.

1. Introduction

The sensitivity of linear dynamic systems has been the subject of many research papers [1–3]. The range of possible uses of the sensitivity analysis is wide and includes such problems as: the approximation of the solutions in the neighbourhood of a known solution, gradient method optimization (including the identification problem) for specified objective functions and the analysis of (measurement) error sensitivity [4]. Generally, the sensitivity problem in practical dynamical cases is so complex that the only viable way of handling it is through computer numerical analysis.

It will be shown here that in a special case, when the dynamical problem is described by differential equations (of any order) with constant coefficients, first and second order sensitivity functions can be determined in an analytical form. The only difficulty is the solution of the characteristic equation. Since in the proposed method only characteristic equation root values are needed, the equation can always be solved numerically (e.g., using the *Mathematica* software).

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The method is applied to the practical problem of railway track vibration, with the intention of using the sensitivity function for the identification of railway track model parameters in the future. In the identification procedure, the particularly interesting parameters are the ones which are unkown and may have an influence on the capacity of the structure. Such design parameters were chosen from a set of all the parameters. The influence of velocity, though the latter was not a design parameter, was also analyzed. This parameter particularly affects the value of railroad displacement and it is easy to control during the identification test. If too many parameters are identified, the error of their identified values can be large. Therefore, the dynamic system identification method requires that the model of the system should not be too complicated.

An infinitely long prismatic beam resting on multiparameter viscoelastic subsoil and loaded with a set of moving forces was assumed as the optimal railway track model. Since the engine moved with a constant velocity, only the stationary problem was considered in the identification procedure. The assumption of stationarity allows a set of moving forces to be substituted for the complicated engine model (a set of sprung and unsprung masses) [5].

The system vibration problem was solved by the Fourier transformation method. The displacement function was obtained in an integral form. Using the residua theorem (the complex function theory) the integral solution was transformed to a closed form. Then by applying theorems concerning the calculation of implicit function derivatives an analytical form of the semilogarithmic (semirelative) sensitivity function was obtained.

The semilogarithmic sensitivity function of quantity w with respect to b was defined by the expression $\partial w/\partial \ln b = b \cdot \partial w/\partial b$ (see also Eqs. (12) and (13) and Refs. [2,5]). Owing to the use of such functions it became possible to study the system's sensitivity to parameter variation for different dimensions and values of the parameters. The used semilogarithmic function indicates what absolute increments of the displacement function will be for the same relative increments of the design parameters. The sensitivity analysis method was tested for all the design parameters and the results for three of them are presented graphically. The *Mathematica* software was used to represent the results as three-dimensional graphs in order to facilitate sensitivity assessment.

2. Problem formulation

An infinite beam resting on a five-parameter elastic subsoil (Fig. 1) and subjected to load $p(x,t) = \sum_j P_j \delta(x - vt - u_j)$ moving with constant velocity v was assumed as the track structure model. This system in a stationary system of co-ordinates x, z is described by the following

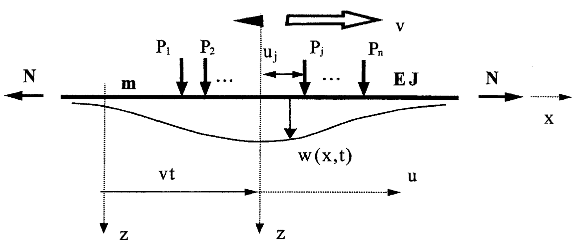

Fig. 1. Model of track structure with load.