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Improved sensitivity analysis by a coupled FE–EFG method

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Abstract

In this paper, we modify the sensitivity analysis of a structural optimization process by using a coupled finite element–element-free Galerkin method. The aim is to improve the sensitivity analysis and to avoid a mesh degradation that can occur when the design variables are perturbed while using classical finite element method. The idea is to replace the finite element mesh by EFGM nodes in areas where the sensitivities of the structural responses have to be computed. The proposed methodology is illustrated by a realistic numerical example for which the finite element method cannot give correct results.

Keywords: Structural optimization; Sensitivity analysis; Element-free Galerkin method; Meshless method

1. Introduction

Over the last few years, numerical simulation has become a tool systematically used in the design phase of industrial parts. The design engineers must analyze complex structures in order to meet ever more drastic specifications. Furthermore, the industrial needs now require optimized mechanical designs.

The optimization of structures has been the subject of specific developments, which allow the engineer to choose the design variables (geometric inputs) and the design constraints (stress or displacement value upper limit...) while performing a finite element simulation.

However, the finite element method is marked by some shortcomings such as discontinuous stress field, need of remeshing in case of severe distortion of the mesh, etc. These shortcomings are even more acute when dealing with shape design optimization problems. In this kind of problems, it is essential to obtain accurate structural responses and their derivatives. In a FEM-based iterative process for improving the design, the need of remeshing to avoid loss of accuracy due to distortion often becomes a necessity and represents a burden in terms of computational time.

Design sensitivity analysis is concerned with finding the variation of a structural response due to a variation of some design parameters, describing the geometry of the domain. The sensitivities are needed in a gradient-based optimization process in order to provide the gradients of the objective function (for example, the area or the mass of a structural part) and of the constraint functions (for example, the admissible stress in the structure) [2]. A review paper in this domain is Haftka and Grandhi [1].

Bobaru et al. developed shape sensitivity analysis in the EFG method context [3]. The derivative of the weak form are computed before discretization. This enables to avoid differentiating the EFG shape functions with respect to the design variables.

Moving least-squares approximation (MLSA) is computationally expensive, since for each integration point, a linear system has to be solved. Moreover, a dense integration pattern is necessary to get accurate values since the EFG method makes use of functions

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that are non rational. Therefore, from the viewpoint of computational time, it is more convenient to use EFG only on the part of the domain where we want to achieve a better approximation of the solution, and to use the FE method for the remaining part of the domain. The coupling between FE and EFG methods has been studied by Belytschko et al. [8] and Hegen [11].

Our research about structural optimization has revealed a main difficulty with the geometry perturbation. Indeed, in order to compute the sensitivity of a structural response with respect to a design variable, the geometry is modified and a new mesh is generated. For most of optimization algorithms, this perturbed mesh must have the same topology as the initial one. This restriction cannot be satisfied if the mesh is generated with the aim of controlling its accuracy. Therefore, we propose to take advantage of the meshless methods. Based on formulations similar to finite element methods, they present the advantage of not requiring a mesh. Hence, they will allow us an easy geometry modification and a more accurate sensitivity analysis. We have demonstrated the applicability of such an approach, first for completely meshless simulations, and then, with the aim of reducing the computational time, for simulations in which the mesh has been deleted in areas where the sensitivities are computed, leaving the finite element mesh elsewhere.

The outline of this paper is as follows: The sensitivity analysis and the usefulness of meshless methods is described in Section 2. Section 3 summarizes the element-free Galerkin method formulation while Section 4 describes a coupled formulation finite element–element-free Galerkin method. A numerical test illustrates our work in Section 5.

2. Sensitivity analysis

Sensitivity analysis consists in computing the first derivatives of the structural responses (displacements, stresses, eigenfrequencies, etc.) with respect to the design variables. This computation is essential because it systematically couples the structural analysis step and the optimization step of the general optimization process. It allows to replace the implicit original optimization problem by a series of approached sub-problems which are explicit (approximations based on the gradients, like the convex linearisation). If the sensitivities are badly estimated, for instance because the mesh is too coarse, the approached sub-problems will be erroneous and the optimization algorithm will not converge to the optimal design. Therefore it seems necessary to improve the accuracy of the sensitivity analysis.

The resolution of an optimization problem by the approximations method requires two kinds of computations:

- a structural analysis by the finite element method in order to obtain the values of the structural responses.
- a sensitivity analysis in order to obtain the first derivatives of these values.

2.1. Definition

The sensitivity of a structural response with respect to a design variable is a kind of material derivative which takes into account the shape modification of the structure due to the parameter perturbation. Let us consider the domain represented on Fig. 1. The initial design correspond to the current value of the design variable X_i, the modified design correspond to the perturbed value $X_i + \delta X_i$.

Let $P(x)$ be a material point in the initial geometry, $P(\bar{x})$ is the same material point in the modified design. The structural response at $P(x)$, for instance the displacement u, can be expressed, in a general way, as a function of the design variable and of the position:

$$u = u(X_i, x(X_i)) \tag{1}$$

The sensitivity of u, with respect to the design variable X_i is given by:

$$\frac{Du}{DX_i} = \frac{\partial u}{\partial X_i} + \sum_{j=1}^{3} \frac{\partial u}{\partial x_j}\frac{\partial x_j}{\partial X_i} \tag{2}$$

Numerically, the finite difference method is a simple and usually used way to compute sensitivities: the structural responses are computed for the current values of the design variables and for the perturbed values. The difference divided by the perturbation of the design variables gives an approximation of the exact derivative. Another way to compute sensitivities is the semi-analytical method: the equations between the structural responses and the generalized displacements are derived analytically.

If we choose to compute (2) by finite differences for instance, several error sources will appear:

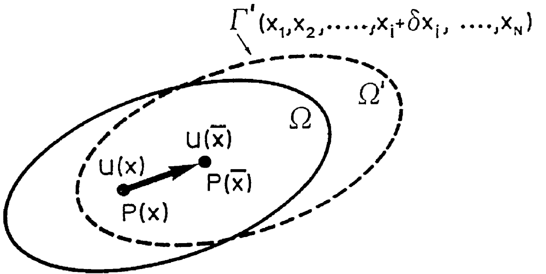

Fig. 1. Initial and perturbed designs.