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An alternative approach for addressing the failure probability-safety factor method with sensitivity analysis

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Abstract

The paper introduces a method for solving the failure probability-safety factor problem for designing engineering works proposed by Castillo et al. that optimizes an objective function subject to the standard geometric and code constraints, and two more sets of constraints that simultaneously guarantee given safety factors and failure probability bounds associated with a given set of failure modes. The method uses the dual variables and is especially convenient to perform a sensitivity analysis, because sensitivities of the objective function and the reliability indices can be obtained with respect to all data values. To this end, the optimization problems are transformed into other equivalent ones, in which the data parameters are converted into artificial variables, and locked to their actual values. In this way, some variables of the associated dual problems become the desired sensitivities. In addition, using the proposed methodology, calibration of codes based on partial safety factors can be done. The method is illustrated by its application to the design of a simple rubble mound breakwater and a bridge crane.

Keywords: Sensitivity analysis; Optimization; Automatic design; Duality

1. Introduction and motivation

Engineering design of structural elements is a complicated and highly iterative process that usually requires a long experience. Iterations consists of a trial-and-error selection of the design variables or parameters, together with a check of the safety and functionality constraints, until reasonable structures, in terms of cost and safety, are obtained.

Optimization procedures are a good solution to free the engineer from the above-mentioned cumbersome iterative process, i.e. to automate the design process [1,2,5,13,18,19].

Safety of structures is the fundamental criterion for design [3,8–10,14–16,20–22]. To this end, the engineer first identifies all failure modes of the work being designed and then establishes the safety constraints to be satisfied by the design variables. To ensure satisfaction of the safety constraints, two approaches are normally used: (a) the classical safety factor approach and (b) the probability-based approach.

With the purpose of illustration, consider the case of designing a breakwater (see Fig. 1) fixing its geometry and dimensions, and checking its behavior with respect to the most important failure modes, as overtopping, overturning and sliding. This check can be done using safety factors, failure probabilities or both. Each failure mode has a probability of occurrence that depends on the selected geometry. A given design must guarantee that the failure probabilities associated with all failure modes are smaller than the values required by the engineering codes. In addition, it is fundamental to choose a design that minimizes the cost.

Classic engineers criticize the probabilistic approach because of its sensitivity to statistical hypotheses, especially tail assumptions [4,11]. Similarly, probability-based engineers question classical designs because it is not clear how far are their designs from failure. To avoid the lack of agreement between defenders of both approaches, and to obtain a more reliable design, Castillo et al. [6,7] proposed

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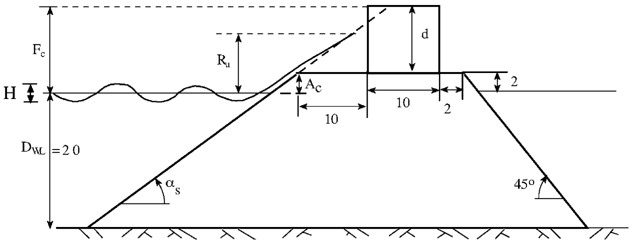

Fig. 1. Parameterized rubblemound breakwater used in the example.

a mixed method, the failure probability-safety factor method (FPSF) that combines safety factors and failure probability constraints.

Since the failure probability bounds cannot be directly imposed in the form of standard constraints, optimization packages cannot deal directly with problems involving them. In fact, failure probability constraints require themselves the solution of other optimization problems.

Fortunately, there are some iterative methods for solving this problem that converge in a few iterations to the optimal solution (see for example Refs. [6,7]). However, since the proposed method consists of a bilevel minimization process, one that minimizes cost and others that calculate the reliability indices, and not all variables are involved in both problems, the final result is that only some sensitivities are obtained. In addition, the method requires the use of a relaxation factor that has to be fixed experimentally. In this paper, an alternative procedure that avoids the relaxation factor and allows to perform a complete sensitivity analysis is presented.

The remaining of this paper is structured as follows. In Section 2 the FPSF method for designing engineering works is presented and the methods proposed by Castillo et al. [6,7] for performing a sensitivity analysis are reviewed. In Section 3 an alternative method optimized to perform a complete sensitivity analysis is presented. In Sections 4 and 5, examples of a breakwater and a bridge girder are given to illustrate the new proposals. Finally, in Section 6 some conclusions are drawn.

2. The failure probability-safety factor design method

It is important and clarifying to classify the set of variables involved in an engineering design problem into the following four subsets:

- $\mathbf{d}$ *Optimization design variables*. They are the design variables which values are to be chosen by the optimization program to optimize the objective function (minimize the cost). Normally, they define the dimensions of the work being design, as width, thickness, height, cross sections, etc.
- $\boldsymbol{\eta}$ *Non-optimization design variables*. They are the set of variables which mean or characteristic values are fixed by the engineer or the code and must be given as data to the optimization program. Some examples are costs, material properties (unit weights, strength, Young modula, etc.), and other geometric dimensions of the work being designed.
- $\boldsymbol{\kappa}$ *Random model parameters*. They are the set of parameters defining the random variability and dependence structure of the variables involved. For example, standard deviations, correlation coefficients, etc.
- $\boldsymbol{\psi}$ *Auxiliary or non-basic variables*. They are auxiliary variables which values can be obtained from the basic variables $\mathbf{d}$ and $\boldsymbol{\eta}$, using some formulas. They are used to facilitate the calculations and the statement of the problem constraints.

Examples of this classification are later given for the breakwater and the bridge crane examples.

Then, the engineering design problem [6,7] can be stated as:

$$\underset{\bar{\mathbf{d}}}{\text{Minimize}}\ c(\bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}) \tag{1}$$

subject to

$$g_i(\bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}) \geq F_i^0; \quad \forall i \in I \tag{2}$$

$$\beta_i(\bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) \geq \beta_i^0; \quad \forall i \in I \tag{3}$$

$$h(\bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}) = \boldsymbol{\psi} \tag{4}$$

$$r_j(\bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}) \leq 0; \quad \forall j \in J \tag{5}$$

where the bars and tildes refer to mean or characteristic values of the variables, $c(\bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}})$ is the objective function to be optimized (cost function), (2) are the limit state equations related to the different failure modes, (3) are constraints that fix the lower bounds on the reliability indices, (4) are the equations that allow obtaining the auxiliary variables $\boldsymbol{\psi}$ from the basic variables $\mathbf{d}$ and $\boldsymbol{\eta}$, and (5) are the geometric or code constraints.

Unfortunately, this problem cannot be solved directly because each of the constraints (3) involve a complicated integral or another optimization problem, i.e.

$$\beta_i(\bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = \underset{\mathbf{d}_i, \boldsymbol{\eta}_i}{\text{Minimum}}\ \beta_i = \sqrt{\sum_{j=1}^{n} z_j^2} \tag{6}$$

subject to

$$g_i(\mathbf{d}_i, \boldsymbol{\eta}_i) = 1 \tag{7}$$

$$T(\mathbf{d}_i, \boldsymbol{\eta}_i; \bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = \mathbf{z} \tag{8}$$

$$h(\mathbf{d}_i, \boldsymbol{\eta}_i) = \boldsymbol{\psi} \tag{9}$$

where $\mathbf{d}_i$ and $\boldsymbol{\eta}_i$ are the design points associated with the design $\mathbf{d}$ and $\boldsymbol{\eta}$ random variables for failure mode i, and $T(\mathbf{d}_i, \boldsymbol{\eta}_i; \bar{\mathbf{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ is the usual transformation (Rosenblatt, Nataf) that converts $\mathbf{d}_i$ and $\boldsymbol{\eta}_i$ into the standard independent normal random variables $\mathbf{z}$.