Available at
www.ElsevierMathematics.com
POWERED BY SCIENCE @ DIRECT•
Applied Mathematics Letters 17 (2004) 225–235

Applied Mathematics Letters
www.elsevier.com/locate/aml

Sensitivity Analysis for Generalized Nonlinear Implicit Quasi-Variational Inclusions

XIE PING DING
Department of Mathematics, Sichuan Normal University
Chengdu, Sichuan, 610066, P.R. China

(Received February 2002; accepted May 2002)

Abstract—In this paper, by using a resolvent operator technique of maximal monotone mappings and the property of a fixed-point set of set-valued contractive mappings, we study the behavior and sensitivity analysis of a solution set for a new class of generalized nonlinear implicit quasi-variational inclusions. Our approach and results are new and generalize many known results in this field.

Keywords—Generalized nonlinear implicit quasi-variational inclusion, Sensitivity analysis, Resolvent operator, Hilbert space.

1. INTRODUCTION

It is well known that variational inequality theory has become a very effective and powerful tool for studying a wide range of problems arising in differential equations, mechanics, contact problems in elasticity, optimization and control problems, management science, operations research, general equilibrium problems in economics and transportation, unilateral, obstacle, moving, etc. A useful and important generalization is called variational inclusions. In 1994, Hassouni and Moudafi [1] used the resolvent operator technique for maximal monotone mapping to study a class of mixed type variational inequalities with single-valued mappings which was called variational inclusions. Adly [2], Ding [3–7], Ding and Lou [8], Huang [9–11], Kazmi [12], Noor [13,14], and Noor, Noor and Rassias [15] have obtained some important extensions and generalizations of the results in [1] from various different directions.

The sensitivity analysis of solutions for variational inequalities has been studied extensively by many authors using quite different methods. By using the projection technique, Dafermos [16], Mukherjee and Verma [17], Noor [18], Yen [19] dealt with the sensitivity analysis for variational inequalities with single-valued mappings. By using the implicit function approach that makes use of so-called normal mappings, Robinson [20] dealt with the sensitivity analysis of solutions for variational inequalities in finite-dimensional spaces. By using resolvent operator technique, Adly [2], Noor and Noor [21], and Agarwal, Cho and Huang [22] study sensitivity analysis for quasi-variational inclusions with single-valued mappings.

This project was supported by the NNSF of China (19871059) and the NSF of Education Department of Sichuan Province of China ([2000] 25).

0893-9659/04/$ - see front matter Typeset by $\mathcal{AMS}$-TEX
doi: 10.1016/S0893-9659(03)00243-X

Recently, by using projection technique and the property of fixed-point set of set-valued contractive mappings, Ding and Lou [23] and Hu [24] study the behavior and sensitivity analysis of solution set for generalized quasi-variational inequalities and generalized mixed quasi-variational inequalities, respectively.

Inspired and motivated by recent research in this field, in this paper, by using resolvent operator technique and the property of fixed-point set of set-valued contractive mappings, we study the behavior and sensitivity analysis of the solution set for a class of generalized nonlinear implicit quasi-variational inclusions. Our results improve and generalize many known results in the field.

2. PRELIMINARIES

Let H be a real Hilbert space with a norm $\|\cdot\|$ and an inner product $\langle\cdot,\cdot\rangle$. Let $C(H)$ denote the family of all nonempty compact subsets of H and $H(\cdot,\cdot)$ denote the Hausdorff metric on $C(H)$ defined by

$$H(A,B)=\max\left\{\sup_{a\in A} d(a,B),\ \sup_{b\in B} d(A,b)\right\},\qquad \forall\, A,B\in C(H),$$

where $d(a,B)=\inf_{b\in B}\|a-b\|:b\in B\}$ and $d(A,b)=\inf_{a\in A}\|a-b\|$.

We now consider the following parametric generalized nonlinear implicit quasi-variational inclusion problem. To this end, let Ω be a nonempty open subset of H in which the parameter λ takes values, $N:H\times H\times\Omega\to H$ and $g,m:H\times\Omega\to H$ be single-valued mappings, and $A,B,C,D,G:H\times\Omega\to C(H)$ be set-valued mappings. Let $M:H\times H\times\Omega\to 2^H$ be a set-valued mapping such that for each given $(z,\lambda)\in H\times\Omega$, $M(\cdot,z,\lambda):H\to 2^H$ is a maximal monotone mapping with $(G(H,\lambda)-m(H,\lambda))\cap \operatorname{dom} M(\cdot,z,\lambda)\neq\emptyset$. Throughout this paper, unless otherwise stated, we will consider the following parametric generalized nonlinear implicit quasi-variational inclusion problem (PGNIQVIP):

$$\begin{aligned}&\text{for each fixed } \lambda\in\Omega,\ \text{find } x(\lambda)\in H, u(\lambda)\in A(x(\lambda),\lambda),\ v(\lambda)\in B(x(\lambda),\lambda),\\ &w(\lambda)\in C(x(\lambda),\lambda),\ z(\lambda)\in D(x(\lambda),\lambda),\ s(\lambda)\in G(x(\lambda),\lambda),\ \text{such that}\\ &0\in M(s(\lambda)-m(w(\lambda),\lambda),z(\lambda),\lambda)+N(u(\lambda),v(\lambda),\lambda).\end{aligned}\tag{2.1}$$

Special Cases

(I) If $G=g:H\times\Omega\to H$ is a single-valued mapping, then the PGNIQVIP (2.1) is equivalent to the following parametric generalized quasi-variational inclusion problem:

$$\begin{aligned}&\text{for each fixed } \lambda\in\Omega,\ \textit{find } x(\lambda)\in H,\ u(\lambda)\in A(x(\lambda),\lambda),\ v(\lambda)\in B(x(\lambda),\lambda),\\ &w(\lambda)\in C(x(\lambda),\lambda),\ z(\lambda)\in D(x(\lambda),\lambda),\ \text{such that}\\ &0\in M(g(x(\lambda),\lambda)-m(w(\lambda),\lambda),z(\lambda),\lambda)+N(u(\lambda),v(\lambda),\lambda).\end{aligned}\tag{2.2}$$

(II) If $m(x,\lambda)=0$ for all $(x,\lambda)\in H\times\Omega$, then problem (2.2) reduces to the following parametric problem:

$$\begin{aligned}&\text{for each fixed } \lambda\in\Omega,\ \text{find } x(\lambda)\in H,\ u(\lambda)\in A(x(\lambda),\lambda),\ v(\lambda)\in B(x(\lambda),\lambda),\\ &z(\lambda)\in D(x(\lambda),\lambda),\ \text{such that}\\ &0\in M(g(x(\lambda),\lambda),z(\lambda),\lambda)+N(u(\lambda),v(\lambda),\lambda).\end{aligned}\tag{2.3}$$

(III) Let $\varphi:H\times H\times\Omega\to\mathbf{R}\cup\{+\infty\}$ be such that for each fixed $(z,\lambda)\in H\times\Omega$, $\varphi(\cdot,z,\lambda)$ is a proper convex lower semicontinuous functional satisfying $G(H,\lambda)\cap\operatorname{dom}(\partial\varphi(\cdot,z,\lambda))\neq\emptyset$ where $\partial\varphi(\cdot,z,\lambda)$ is the subdifferential of $\varphi(\cdot,z,\lambda)$. By [25], $\partial\varphi(\cdot,z,\lambda):H\to 2^H$ is a maximal monotone mapping. Let $M(\cdot,z,\lambda)=\partial\varphi(\cdot,z,\lambda)$, $\forall\ (z,\lambda)\in H\times\Omega$. For given $(z,\lambda)\in H\times\Omega$, by the