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Sensitivity analysis of the optimal management policy for a queuing system with a removable and non-reliable server☆

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Abstract

The management policy of an M/G/1 queue with a single removable and non-reliable server is considered. The decision-maker can turn the single server on at any arrival epoch or off at any service completion. It is assumed that the server breaks down according to a Poisson process and the repair time has a general distribution. Arrivals form a Poisson process and service times are generally distributed. In this paper, we consider a practical problem applying such a model. We use the analytic results of the queueing model and apply an efficient Matlab program to calculate the optimal threshold of management policy and some system characteristics. Analytical results for sensitivity analysis are obtained. We carry out extensive numerical computations for illustration purposes. An application example is presented to display how the Matlab program could be used. The research is useful to the analyst for making reliable decisions to manage the referred queueing system.

Keywords: Analytical results; Management policy; Non-reliable; Sensitivity analysis

1. Introduction

In this paper we study the operational characteristics of an M/G/1 queueing system in which a removable and non-reliable server operates with an N policy. The term 'removable server' is just an abbreviation for the system of turning on and turning off the server, depending on the number of

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customers in the system. A non-reliable server means that the server is typically subject to unpredictable breakdowns. The server is removable and applies the N policy: turn the server on whenever N ($N \geq 1$) or more customers are present, turn the server off only when no customers are present. After the server is turned off, the server may not operate until N customers are present in the system.

For a reliable server, the N policy M/M/1 queueing system was first developed by Yadin and Naor (1963), and the N policy M/G/1 queueing system was developed be several researchers such as Bell (1971, 1972), Heyman (1968), Kimura (1981), Teghem (1987), Tijms (1986), Artalejo (1998), and Wang and Ke (2000). For a non-reliable server, Avi-Itzhak and Naor (1963) studied the ordinary M/M/1 queueing system where the service rule does not depend on the number of customers in the queue. The ordinary $M/E_k/1$ queueing system with arrival rate depending on server breakdowns, was investigated by Shogan (1979). Neuts and Lucanton (1979) studied a Markovian queueing system with multiple servers subject to breakdowns and repairs. The explicit solutions for the N policy Markovian queueing systems with a non-reliable server may be used to obtain the results for the N policy M/M/1 queueing system with a reliable server (see Sivazlian & Stanfel, 1975), or the ordinary M/M/1 queueing system with a non-reliable server (see Wang, 1990), or the ordinary M/M/1 queueing system with a reliable server (see Sivazlian & Stanfel, 1975) as a special case.

The purpose of this paper is threefold. First, an efficient Matlab program is used to calculate the optimal policy value N and some system characteristics. Second, the analytical results of the sensitivity analysis are derived. We then carry out extensive numerical computation for sensitivity analysis purpose. Third, we present an application example showing the way in which the Matlab program is used to calculate system characteristics, the optimum value of N and its minimum expected cost for various system parameters, while maintaining the maximum service quality.

Note that existing research works for the queueing system have never investigated the analytic solutions for the sensitivity analysis. In this paper, we will completely and successfully perform the sensitivity analysis for the M/G/1 queueing system with a removable and non-reliable server. Through this sensitivity analysis, we will be able to analyze the complex but exact solutions for a practical and general queueing system.

2. The queueing service model

2.1. System description and assumptions

Referring to Wang and Ke (2002), we consider the following model formulation. A cycle of the model consists of an idle period and a completion period. The completion period is composed into busy period and the breakdown period. As the system is empty, one cycle begins. The server is turned off until there are N customers in the system. We call this the idle period. The busy period is initiated when the server starts serving the waiting customers. While providing the service, the server may break down and be sent for repair immediately. This is called the breakdown period. As soon as the server is repaired, he returns to service again until all customers in the system are serviced. Since the completion period starts when the idle period is over and terminates when there are no customers in the system, the completion period may be represented as the sum of the busy period and the breakdown period. In addition, we consider the model under the following assumptions: