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# Application of BEM and sensitivity analysis to the solution of the governing diffusion–convection equation for a continuous casting process

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## Abstract

This paper presents a two-dimensional diffusion–convection boundary problem which models a continuous casting process of a pure substance (e.g. copper). The boundary problem is formulated for both liquid and solid subdomains separated by a phase change front. Its location is unknown and has to be updated during the iteration process utilising the so-called front tracking algorithm. The solution procedure is based on the Boundary Element Method and involves the total derivative of the temperature and relevant sensitivity coefficients to find a correct position of the phase change front. In order to considerably reduce the number of degrees of freedom in the front tracking algorithm, the position and shape of the phase change front is interpolated by a small number of Bezier segments. Numerical examples are included to demonstrate the main features of the developed algorithms.

*Keywords:* Boundary element method; Diffusion–convection; Continuous casting; Sensitivity coefficients

## 1. Introduction

This paper presents a BEM algorithm applicable to the solution of the steady-state diffusion–convection equation which governs the numerous physical problems, including the heat transfer problem in continuous casting processes. Since the process of the continuous casting is nowadays frequently utilised in the metallurgical industry and in materials engineering in general, the transport phenomena encountered in this solid–liquid phase change system are of great importance for fundamental research, as well as for engineering and technological practice.

The continuous casting process is typically organised that the liquid material (e.g. metal) is poured into the mould whose walls are cooled by the flowing water. Consequently, the whole system disperses the heat and the liquid phase solidifies. The side surface of the obtained ingot is still very intensively cooled, usually by water flowing out of the mould and being sprayed over the surface. The cooled ingot is pulled out by withdrawal rolls at a constant rate. Details of the technological process, only mentioned here, can be found elsewhere [1].

In reality, the physical processes which take place in this phase change system are of a very complex nature and difficult to be analysed. Taking into account all computational difficulties at the stage of determining the temperature field, which is the subject of this work, the numerical methods are almost exclusively used. Furthermore, the position of the solid–liquid interface is generally unknown and has to be determined together with the temperature distribution and heat fluxes. Because of the number of nonlinearities, calculations are carried out iteratively.

In general, there are two main approaches to the solution of this kind of boundary problems, i.e. the fixed domain method and the front tracking method. The majority of the formulations based on the fixed domain method use a quantity known as enthalpy. The phase change condition is actually accounted implicitly without attempting it a priori to establish the position of the front. As a result the position

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of the phase change front is calculated in the post-processing operations. Introducing in addition a quantity called *the liquid fraction*, which in principle has to be determined at each volume numerical element within computational domain, both solidus and liquidus can be found.

In the front tracking technique the phase change front is accounted for explicitly and searching for its unknown position is the essential part of the solution. Typically, the computational domain is divided into two subdomains, i.e. liquid and solid regions, separated by the phase change front where the so-called Stefan condition is prescribed. There is an extensive literature on the Stefan problem and the position of the front can only be found iteratively, updating the numerical mesh each time when the front is moved.

Both the fixed domain method and the front tracking method can involve any general purpose numerical technique, i.e. finite-difference method, finite element method and/or boundary element method. A fixed domain approach with the Finite Element Method can be found in Refs. [2,3]. The least squares approach to front tracking and Finite Element Method in two-dimensional Stefan problems is extensively discussed in Ref. [4].

It should be stressed that the application of the Boundary Element Method for the considered boundary problem reduces the discretization to the boundary only and updating of the position of the phase change front involves only the remeshing of the front itself. This feature makes the calculations quite straightforward and easier compared to other numerical methods. Additionally, heat fluxes playing an important role in the Stefan condition are generally modelled much more accurately than in FDM or FEM.

Application of BEM can be found in many works [5,6], where different solutions for continuous casting are considered. In Ref. [5] the front tracking approach is applied and the position of the phase change front was searched using an optimisation technique and quadratic elements. In Ref. [6] the results of BEM fixed grid approach are compared with results obtained by the front tracking technique.

This paper presents advances in the BEM algorithms applicable to the solution of the diffusion–convection equation which governs the steady-state heat transfer problem in the continuous casting process. The iterative front tracking solution procedure starts from the assumption of an initial position of the phase change front. For such a defined geometry of the system, the heat transfer problem is solved by BEM, forcing continuity and compatibility conditions along the phase change interface. This situation is quite similar to the case of a domain composed of subdomains and classical BEM computer code is applicable. However, it should be noted that the treatment of the compatibility condition requires minor modifications resulting from the fact that heat flux has a known jump on the interface equal to the latent heat [5]. Obtaining the temperature along the front (which is certainly different from the phase change temperature) one can now search for the new location of the front using the so-called sensitivity coefficients. Obtaining the new geometry of the analysed system, the whole procedure of determining the temperature field, calculating the sensitivity coefficients and updating the position of the front can be repeated until the convergence criterion is satisfied. In this work calculations stop when the temperature along the phase change front is sufficiently close to the melting temperature.

One of the most important elements of the above algorithm is the front tracking algorithm and the way the position and shape of the phase change front is represented numerically. From the previous works [5], it was concluded that the phase change boundary should possess $C^1$ continuity. Such requirement restricts the shape of the front to quadratic or higher-order elements. So far quadratic elements were mainly used [5,7]. Since the number of variables used to describe the shape and position of the interface is in this case quite large (i.e. all nodes lying on the front), in Ref. [8] the authors proposed the use of cubic elements and span them along a moderate number of Bezier splines. It was shown that only 2–4 Bezier splines are necessary for a reasonable approximation of the phase change front. This means that the phase change interface is characterised by a significantly fewer number of variables (so-called control points [9]) than in the case of using quadratic elements. The same idea is used in the present work.

Together with utilising Bezier segments, a novel front tracking algorithm is proposed in this paper. The new position of the phase change front is found by minimising an appropriate objective function and utilising the total derivative of the temperature and relevant sensitivity coefficients. The objective function is defined as a square of the temperature differences from the melting point, summed over all the nodes lying on the phase change front. In the numerical examples carried out so far, a variety of different casting parameters (boundary conditions, casting velocities, etc.) have been tested. It was proved that the proposed approach reduces considerably the number of iterations and at the same time this guarantees good accuracy and overall the high efficiency of the algorithm. In Ref. [10] results were verified by comparing them with results obtained using the enthalpy formulation and the finite volume method. It was found that the proposed algorithm gives almost the same results as those obtained using the commercial package Fluent [11] but they were obtained within significantly shorter time. Small differences were noticed only for materials with very low thermal conductivity and latent heat (e.g. lead).

It must also be stressed that the total derivatives of the temperature, necessary for the calculation of the new position of the phase change front within iteration process, can be either obtained in a simplified way, i.e. displacing each control point in succession and applying an appropriate