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Sensitivity analysis in investment project evaluation

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Abstract

This paper discusses the sensitivity analysis of valuation equations used in investment decisions. Since financial decision are commonly supported via a point value of some criterion of economic relevance (net present value, economic value added, internal rate of return, etc.), we focus on local sensitivity analysis. In particular, we present the differential importance measure (DIM) and discuss its relation to elasticity and other local sensitivity analysis techniques in the context of discounted cash flow valuation models. We present general results of the net present value and internal rate of return sensitivity on changes in the cash flows. Specific results are obtained for a valuation model of projects under severe survival risk used in the industry sector of power generation.

Keywords: Investment evaluation; Sensitivity analysis

1. Introduction

In this paper, we discuss the sensitivity analysis (SA) of valuation equations used in investment project valuation. Investment project decision making involves the use of valuation models that require the estimation of the investment cash flows (CF), that feed into the equation of the economic criterion (net present value—"NPV"—or internal rate of return—"IRR") that supports the decision (Taggart, 1996). We denote the valuation criterion as Y, and write

$$Y = f(\mathbf{x}), \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is the set of the model input parameters.

The decision often relies on the value of the criterion that is obtained when the input parameters are fixed at the so-called base-case values. Such values reflect the analyst/decision maker knowledge of the investment assumptions. We write

$$Y^0 = f(\mathbf{x}^0), \tag{2}$$

where $\mathbf{x}^0$ denotes the base-case value of the parameters.

The decision making process is integrated by SA, where the analyst assesses the effect on Y^0 of changes in the input parameters. Such sensitivity is usually tested through, what is called, a "one way" or a combined SA scheme, where the analyst registers the change in Y^0, $\Delta Y = Y - Y^0$, obtained when a parameter or a combination of parameters is varied by a rationally assumed range. Such analysis is used to draw conclusions

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on the consistency and correctness of the valuation model, as well as, in some cases, it integrates risk analysis giving information on the investment risk consequent to changes in the assumptions. The decision maker is often lead by this analysis to infer a relative importance of the parameters/assumptions and to rank them according to their influence on Y^0 (the classical "Tornado diagram" scheme (Clemen, 1998)). However, while this SA scheme is appropriate for the first and second task, it should not be used to infer parameter importance, since the parameter changes (Δx_i) are not taken into consideration (Borgonovo and Apostolakis, 2001a,b; Borgonovo, 2001b).

Several local SA techniques have been recently developed in the literature to infer parameter importance (Borgonovo and Apostolakis, 2001a; Borgonovo, 2001b; Cheok et al., 1998; Helton, 1993; Turanyi and Rabitz, 2000; Koltai and Terlaky, 2000). We focus on the differential importance measure (DIM), a SA technique recently proposed (Borgonovo, 2001a; Borgonovo and Apostolakis, 2001a,b). We highlight its relationship to other local SA techniques, with particular reference to elasticity. We consider the application of DIM on NPV valuation equations. We show that through DIM the analyst is able to obtain the importance of any arbitrary change in the magnitude of the CFs. Besides, the importance of the CFs for NPV equations has an intuitive interpretation, which we will discuss. We, then, focus on the SA of IRR equations.

We apply the results obtained for NPV and IRR equations to the local SA of a specific model, developed for the evaluation of projects under survival risk and used in the energy sector (Beccacece et al., 2000).

In Section 2, we present the definition of DIM, its computation and its relationship with other local SA techniques. In Section 3, we discuss the application of DIM to NPV and IRR valuation equations. In Section 4, we present an overview of the model for project evaluation, we discuss application and results of the local SA of the model through DIM and elasticity. Conclusions are offered in Section 5.

2. Local sensitivity analysis techniques: The differential importance measure

In the recent past, the activity in the scientific field of SA of model output has been steadily growing. Due to the increasing complexity of numerical models, SA has acquired a key role in testing the correctness and corroborating the robustness of models in several disciplines. This has led to the development and application of several new SA techniques (Borgonovo and Apostolakis, 2001a; Helton, 1993; Saltelli, 1997, 1999; Saltelli and Marivoet, 1990; Saltelli et al., 1999; Turanyi and Rabitz, 2000; Koltai and Terlaky, 2000). Since financial decisions in investment valuation are often based on the nominal value of the project economics chosen as valuation criterion (NPV, IRR etc.), we will focus on local SA techniques. The next paragraphs present the differential importance measure (DIM) and its relation to other local SA techniques.

The framework for the utilization of DIM is set forth by Eqs. (1)–(2). The definition of DIM can be stated starting from $\mathbf{x}^0$ and considering the increment vector:

$$\mathrm{d}\mathbf{x} = [\mathrm{d}x_1, \mathrm{d}x_2, \ldots, \mathrm{d}x_n]^{\mathrm{T}}.$$

The DIM for the input x_s is defined as follows (Borgonovo and Apostolakis, 2001a):

$$D_s(\mathbf{x}^0, \mathrm{d}\mathbf{x}) = \frac{f'_{x_s}(\mathbf{x}^0)\,\mathrm{d}x_s}{\sum_{j=1}^{n} f'_{x_j}(\mathbf{x}^0)\,\mathrm{d}x_j} = \frac{\mathrm{d}f_s(\mathbf{x}^0)}{\mathrm{d}f(\mathbf{x}^0)}. \tag{3}$$

The requirements under which D_s is defined are that $Y = f(\mathbf{x})$ is a criterion function differentiable at $\mathbf{x}^0$ and $\mathrm{d}\mathbf{x}$ is not orthogonal to the gradient of Y at $\mathbf{x}^0$, i.e. $\mathrm{d}f(\mathbf{x}^0) \neq 0$. $D_s(\mathbf{x}^0, \mathrm{d}\mathbf{x})$ is the ratio of the partial differential of Y w.r.t. x_s and its total differential, both computed on the basis of $\mathrm{d}\mathbf{x}$. DIM produces the importance of parameters for small deviations of their value from the base-case value.

It is worth mentioning the following two properties of DIM:

- D_s shares the additivity property with respect to the various inputs, i.e. the DIM of some set of parameters coincides with the sum of the individual DIMs of the parameters in that set