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Sensitivity analysis of non-conservative eigensystems

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Abstract

An expression for the derivatives of eigenvalues and eigenvectors of non-conservative systems is presented. Contrary to previous methods that use state space form ($2N$-space) to consider damping, proposed method solves the eigenpair derivatives of damped system explicitly. The computation size of N-order is maintained and the eigenpair derivatives are obtained simultaneously from one equation so that it is efficient in CPU time and storage capacity. Moreover, this method can be extended to asymmetric non-conservative damped systems. Although additional problems are generated contrary to the eigenpair sensitivity methods of symmetric systems, in asymmetric case, an algebraic method for the eigenpair derivatives can be obtained through similar procedure. The proposed expression is derived by combining the differentiations of the eigenvalue problem and normalization condition into one linear algebraic equation. The numerical stability is proved by showing non-singularity of the proposed equation, and the efficiency of the derived expression is illustrated by considering a cantilever beam with lumped dampers and a whirling beam.

1. Introduction

Natural frequencies and mode shapes of systems are essential to understand dynamic behavior of structure. However, design parameters can be varied with damage, deterioration, corrosion, etc. and this causes variation in natural frequency and mode shape. The variation of eigenpair brings about variation of dynamic behavior of systems and this affects the stability of structure

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directly. Therefore, eigen-sensitivity analysis has played a central part in structural stability analysis and has emerged as an important area of research. And eigenpair sensitivity is used in many areas, the optimization of structure subject to natural frequency, system identification, finite modelling updating, structural control, etc.

A number of methods for eigenpair sensitivity of undamped system have been developed. Fox and Kapoor [1] finded the eigenpair derivatives with the term of system matrix and eigenpair. Nelson [2] represented eigenvector derivative by sum of the homogeneous solution and the particular solution, and Ojalvo [3] and Dailey [4] extended Nelson's method to the multiple eigenvalue problem. Modal method [5,6] and its modified one [7,8] approximated the mode shape derivatives by the linear combination of mode shapes, and Lee and Jung [9,10] presented the algebraic methods for eigenpair derivatives of systems having the distinct and multiple eigenvalue.

A number of the prescribed methods can be applied to the damped systems. However, almost eigen-sensitivity methods have to use state space equation based on $2N$-space to solve the problems induced by damping. These methods are at a disadvantage in CPU time and storage capacity because of double computation size. In order to overcome these drawbacks, Zimoch [11] presented direct method for the eigenpair derivatives of damped systems without use of state space equation. However, this method is restricted to mechanical systems because the available design parameter is limited to the component of the system matrices. Sodipon Adhikari [12] proposed eigen-sensitivity method based on N-space, too. However, it did not give exact solution and only is applicable to small sized damped systems. On the other hand, Lee et al. [13,14] developed analytical method that give exact solutions while it maintain 'N-space', but it finds eigenvalue derivative from classical method as before.

Many eigenpair sensitivity methods are restricted to systems whose characteristic matrices are symmetric. However, a number of real systems have asymmetric mass, damping, and stiffness matrices, for example, the behavior of structure in fluid, moving vehicles on roads, the study of aircraft flutter and gyroscopic systems. It is difficult to solve the eigenpair sensitivity of asymmetric systems by using the previous methods because of additional problems due to asymmetric characteristic matrices. And this difficulty is possibly motivation for authors that have tried to solve the eigenpair sensitivity of asymmetric systems.

Fox and Kapoor [1] presented exact expression for eigenpair derivatives of symmetric undamped systems in the earliest time and many authors [15–17] have extended his method [1] to asymmetric systems. Rudisill [18] solved the eigenvector derivatives of general matrices analytically and Murthy and Haftka [19] have written an excellent review on calculating the eigenpair derivatives of general matrices.

However, above methods don't explicitly consider the damping of systems. Brandon [20] presented the modal method for asymmetric damped systems. This method solved the problems due to asymmetric matrices by using the left eigenvector. However, it has disadvantages in CPU time and storage capacity because it uses state space form to consider damping of systems and requires many of eigenpair information to find eigenpair sensitivity.

In this paper, an efficient algebraic method for the eigenpair sensitivities of damped systems is presented. Contrary to previous methods the proposed method finds the eigenvalue and eigenvector sensitivities simultaneously from one equation. And the proposed method does not use state space equation ($2N$-space), instead of it, the method maintain 'N-space' because singularity problem is solved by using only a side condition. The proposed method gives exact