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Sensitivity analysis and optimization of thermo-elasto-plastic processes with applications to welding side heater design

J. Song [a,1], J.Y. Shanghvi [a,1], P. Michaleris [b,*]

[a] *Department of Mechanical and Nuclear Engineering, 137 Reber Building, Pennsylvania State University, University Park, PA 16802, USA*

[b] *Department of Mechanical and Nuclear Engineering, 232 Reber Building, Pennsylvania State University, University Park, PA 16802, USA*

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Abstract

A computational scheme is presented to optimize quasi-static weakly coupled thermo-elasto-plastic processes in three dimensional Lagrangian reference frames. Sensitivity formulations are developed from the radial return algorithm based thermo-elasto-plastic finite element equations using the direct differentiation method. These formulations are exemplified in the optimization of the side heaters in the transient thermal tensioning welding process for minimum residual stress. The results of the direct differentiation sensitivity analysis are validated by comparing with finite difference sensitivity calculations. Optimization is performed using the BFGS line search method.

Keywords: Thermo-elasto-plasticity; Radial return algorithm; Non-linear finite element analysis; Lagrangian frame; Sensitivity; Optimization; Welding; Side heater

1. Introduction

In material processes such as welding and laser forming, the resultant permanent transformations of the materials are dependent on design variables such as heat input and travel speed. An empirical approach to optimize material processes is time consuming and costly. Therefore, computational optimization methodologies are required.

For computationally intensive problems, gradient optimization methods are computationally efficient. However, they require the evaluation of sensitivities of the solution with respect to each design variable.

* Corresponding author. Tel.: +1-814-863-7273; fax: +1-814-863-4848.
E-mail addresses: jxs909@psu.edu (J. Song), janaks@psu.edu (J.Y. Shanghvi), pxm32@psu.edu (P. Michaleris).
[1] Tel.: +1-814-865-0059.

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Sensitivity analysis has been widely used in many design optimization problems [1–8]. Sensitivity analysis can be performed by analytical or by finite difference techniques [9]. The analytical methods are more accurate and computationally more efficient than the finite difference method. The analytical sensitivities can be computed either by direct differentiation or by the adjoint method [2]. For transient problems, the direct differentiation method is algorithmically more efficient than the adjoint method.

Sensitivity analysis for coupled systems is presented in Ref. [10]. Sensitivity analysis of thermo-elasto-plastic processes in two dimensional frames with the assumption of generalized plane strain has been implemented in minimizing welding residual stress and distortion in Ref. [11]. Sensitivity analysis for thermo-elasto-plastic processes in Eulerian reference frames has been developed in optimizing the laser forming process in Ref. [12]. These two approaches have critical assumptions in their formulations. The two dimensional approach is limited in accounting for three dimensional effects and the Eulerian approach is applicable only for steady-state processes. Michaleris et al. [13] have demonstrated a sensitivity analysis for thermo-elasto-plastic processes in Lagrangian reference frames. However, they use a stress update algorithm (fully implicit backward Euler in tensor form) that is computationally less efficient than the radial return algorithm [14] and, thus, the sensitivity algorithm is implemented only for a two dimensional example in their paper. Therefore, it is necessary to develop sensitivity equations for thermo-elasto-plastic processes with the radial return algorithm in three dimensional Lagrangian reference frames.

In this paper, conventional finite element formulations for thermo-elasto-plastic analyses with the radial return algorithm in three dimensional Lagrangian reference frames are reviewed. Then, thermo-elasto-plastic sensitivity equations are developed using the direct differentiation method. The sensitivity formulations are verified by comparing with results obtained by the finite difference sensitivity analysis. The sensitivity equations are implemented to a welding optimization example. The positions and the heat input power of side heaters in the fillet welding process are optimized for minimum residual stress.

2. Review of finite element formulations for thermo-elasto-plastic processes

Finite element formulations for quasi-static thermo-elasto-plastic processes in Lagrangian reference frames have been widely used [13–21]. The thermal analysis is assumed to be transient while the elasto-plastic quasi-static. Many thermo-elasto-plastic processes such as welding and thermally assisted forming are typically assumed to be weakly coupled, that is, the temperature profile is assumed to be independent of stresses and strains. Thus, a heat transfer analysis is performed initially and the temperature history is imported as loading in the mechanical analysis. Both thermal and mechanical problems are non-linear due to temperature-dependent material properties and plasticity, respectively.

2.1. Transient thermal analysis

For a Lagrangian coordinate $\mathbf{x}$ and time t, the governing equation for transient heat conduction analysis is given as follows:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot [\mathbf{k} \nabla T] + Q \text{ in the entire volume } V \text{ of the material,} \tag{1}$$

where ρ is the density of the body, C_p is the specific heat capacity, T is the temperature, $\mathbf{k}$ is the temperature-dependent thermal conductivity matrix, Q is the internal heat generation rate, and ∇ is the Lagrangian gradient operator.