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FEM–BEM-coupling and structural–acoustic sensitivity analysis for shell geometries

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Abstract

Passive noise control by modification of structural geometry moves more and more into the field of vision for designers. This structural–acoustic optimization shows high potential in minimization of radiated noise especially for thin shell geometries. Since computer systems have been enhanced dramatically in recent years numerical simulation of structural vibration and acoustic field was accelerated significantly. But still the sensitivity analysis represents the bottle neck in computational efforts of gradient-based optimization. Furthermore, the coupling between FE and BE analysis in structural–acoustic simulation of large scale models will occur as an additional challenge because of non-matching meshes. In this paper, a fast method for computation of sensitivities of acoustic properties with respect to shell geometry is presented. Therefore, a concept to parameterize the structural geometry is explained. Additionally, the coupling procedure is simplified to reduce computational effort. Finally, this method is applied to an academic example. The sensitivity of the sound pressure at an internal point of an oblique box is investigated, when one side of this six-sided box is geometrically modified.

Keywords: Acoustic optimization; FEM–BEM-coupling; Fluid–structure-interaction; Sensitivity analysis; Shape optimization; Gradient-based optimization; Noise control

1. Introduction

Since acoustic properties of components became an additional challenge in design, efficient techniques may be sought within numerical optimization processes. Due to improved computational capabilities in recent years the simulation of acoustic fields in accordance to vibrating and thus sound radiating structures has significantly accelerated. An acoustic property can be minimized by modifying design variables of the structure during the optimization. Marburg [1] presents an overview of the current state-of-the-art in structural–acoustic optimization. Furthermore, structural–acoustic sensitivities with respect to design variables are most expensively computed in gradient-based optimization. Often, these sensitivities are approximated using global finite-difference schemes for the objective function, cf. Lamancusa [2], Christensen et al. [3], Marburg and Hardtke [4]. This method requires at least one additional computation of objective function for each design parameter. To overcome this problem, semi-analytical or even analytical sensitivity analysis appears more appropriate. Thus, adjoint variable methods as in Choi et al. [5,6]

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or Wang et al. [7,8] can be implemented. Generally, these applications need to compute the acoustic field for each design step. By simplifying the fluid–structure coupling to a purely structure-induced noise the acoustic model is supposedly negligibly changed by structural modification. This is suitable for structures being more dense than the fluid. This approach represents the structural dynamic behavior in vacuum followed by a post-processing step to evaluate the acoustic properties. The acoustic sensitivity with respect to surface velocity used in this method may be termed as influence coefficients, cf. Ishiyama et al. [9], Marburg [10], or recently as acoustic transfer vectors [11]. Usually, structural frequency response is calculated for broad frequency ranges by FEA using mode superposition techniques. To prevent discretizing the three-dimensional fluid, we carry out a boundary element analysis to compute sound pressures at internal points of the cavity. These two methods do not necessarily need to be applied on the same mesh. Thus, a coupling procedure must correlate structure to fluid mesh and vice versa. Parameterizing the geometric modification of the shell structure will reduce the number of design variables and the optimization procedure becomes more manageable. All these concepts are applied in an example of an oblique six-sided box to avoid effects by symmetry. Herein the sound pressure at an internal point of the box is optimized by modifying one side of the box using a spline parameterization concept.

2. Structural–acoustic simulation

2.1. Structural dynamics

It is assumed hereon that all field variables behave stationary. Hence, we apply a steady-state approach separating time-dependent variables $\widetilde{F}(t)$ at field point $\vec{x}$ into a complex frequency-dependent spatial part f and a transitory function $\exp(\mathrm{i}\Omega t)$ with Ω being the circular frequency

$$\widetilde{F}(\vec{x},t) = \Re\{f(\vec{x},\Omega)\mathrm{e}^{\mathrm{i}\Omega t}\}. \tag{2.1}$$

For simulation of the dynamic behaviour of shell structures we follow the shell formulation by Naghdi. By using finite-element discretization for all components u_i of the spatial structural displacement vector $\bar{\boldsymbol{u}}$ yields

$$\bar{\boldsymbol{u}}(\vec{x},\Omega) = [\,u_1 \quad u_2 \quad u_3\,]^{\mathrm{T}} = \boldsymbol{N}_{\mathrm{s}}(\vec{x})\boldsymbol{u}(\Omega), \tag{2.2}$$

wherein $\boldsymbol{N}_{\mathrm{s}}$ represents the matrix of nodal interpolation functions and $\boldsymbol{u}$ contains the nodal displacements. Following the derivation of finite-element matrices [12] the linear system of equations to compute the nodal displacements $\boldsymbol{u}$ is obtained

$$\boldsymbol{A}(\Omega)\boldsymbol{u}(\Omega) = (\boldsymbol{K} + \mathrm{i}\Omega\boldsymbol{B} - \Omega^2\boldsymbol{M})\boldsymbol{u}(\Omega) = \boldsymbol{r}(\Omega). \tag{2.3}$$

In (2.3) the global system matrix $\boldsymbol{A}$ is assembled by the stiffness matrix $\boldsymbol{K}$, the viscous damping matrix $\boldsymbol{B}$ and the mass matrix $\boldsymbol{M}$. Furthermore, the system of equations is completed by the right hand side vector $\boldsymbol{r}$ representing the excitation of the structure. Denoting the degrees of freedom of the system by n_{s} a $n_{\mathrm{s}} \times n_{\mathrm{s}}$ matrix $\boldsymbol{A}$ is produced. In general this structural system of equations will be solved by inversion of $\boldsymbol{A}$. Since we consider very large structural systems with low damping an approximation approach of the system inverse $\boldsymbol{A}^{-1}(\Omega)$ will be more suitable for frequency bands. Therefore, the general real eigenvalue problem is solved

$$(\boldsymbol{K} - \omega_i^2\boldsymbol{M})\boldsymbol{y}_i = \boldsymbol{0} \quad \text{and} \quad \boldsymbol{V} = [\,\boldsymbol{y}_1 \quad \boldsymbol{y}_2 \quad \cdots \quad \boldsymbol{y}_{n_{\mathrm{ev}}}\,] \tag{2.4}$$

to compute the eigenvalues ω_i^2 and their corresponding eigenvectors $\boldsymbol{y}_i$. The eigenvectors $\boldsymbol{y}_i$ are stored in the modal matrix $\boldsymbol{V}$ up to the number n_{ev} of calculated eigenvalues. This number is chosen with respect to the investigated frequency range and represents a modal reduction approach, cf. Gasch and Knothe [13]. The eigenvectors in $\boldsymbol{V}$ are furthermore normalized by the orthogonalization condition $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{V} = \boldsymbol{I}$. Using (2.4) and the orthogonalization condition for (2.3) the system inverse is approximated by modal superposition as

$$\boldsymbol{A}^{-1}(\Omega) \approx \boldsymbol{V}\boldsymbol{\Delta}(\Omega)\boldsymbol{V}^{\mathrm{T}}. \tag{2.5}$$

The $n_{\mathrm{ev}} \times n_{\mathrm{ev}}$ inverted modal system matrix $\boldsymbol{\Delta}$ given by

$$\boldsymbol{\Delta} = [\boldsymbol{V}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{V} + \mathrm{i}\Omega\boldsymbol{V}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{V} - \Omega^2\boldsymbol{I}]^{-1} \tag{2.6}$$

contains the modal stiffness matrix $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{V}$ and the modal damping matrix $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{V}$. For certain damping approaches other than employed in (2.3) it is not necessarily possible to diagonalize these matrices. The nodal displacement vector here can be written as

$$\mathbf{u}(\Omega) = \boldsymbol{A}^{-1}(\Omega)\mathbf{r}(\Omega) \approx \boldsymbol{V}\boldsymbol{\Delta}(\Omega)\boldsymbol{V}^{\mathrm{T}}\boldsymbol{r}(\Omega). \tag{2.7}$$

Although the modal system matrix still has to be inverted at each frequency a much smaller $n_{\mathrm{ev}} \times n_{\mathrm{ev}}$ matrix is handled rather than the global system matrix $\boldsymbol{A}$.

2.2. Acoustic field simulation

Keeping the steady-state approach in (2.1) the basic differential equation for the sound pressure p to describe linear acoustics [14] is given by the Helmholtz equation

$$\Delta p(\vec{x},\Omega) + k^2 p(\vec{x},\Omega) = 0 \quad \text{with } k = \frac{\Omega}{c}, \tag{2.8}$$

where the wave number k is determined by the circular frequency Ω and the speed of sound c. This boundary value problem requires boundary conditions which are, for example, given by a Robin boundary condition