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Finite element response sensitivity analysis: a comparison between force-based and displacement-based frame element models

M. Barbato, J.P. Conte *

Department of Structural Engineering, University of California at San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0085, United States

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Abstract

This paper focuses on a comparison between displacement-based and force-based elements for static and dynamic response sensitivity analysis of frame type structures. Previous research has shown that force-based frame elements are superior to classical displacement-based elements enabling, at no significant additional computational costs, a drastic reduction in the number of elements required for a given level of accuracy in the simulated response. The present work shows that this advantage of force-based over displacement-based elements is even more conspicuous in the context of gradient-based optimization methods, which are used in several structural engineering sub-fields (e.g., structural optimization, structural reliability analysis, finite element model updating) and which require accurate and efficient computation of structural response and response sensitivities to material and loading parameters. The two methodologies for displacement-based and force-based element sensitivity computations are compared. Three application examples are presented to illustrate the conclusions. Material-only non-linearity is considered. Significant benefits are found in using force-based frame element models for both response and response sensitivity analysis in terms of trade-off between accuracy and computational cost.

Keywords: Plasticity-based finite element models; Response sensitivity analysis; Force-based frame elements

* Corresponding author.
E-mail address: jpconte@ucsd.edu (J.P. Conte).

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1. Introduction

In recent years, great advances in the non-linear analysis of frame structures were led by the development of force-based elements, which have been found superior to classical displacement-based elements in tracing material non-linearities such as those encountered in steel, reinforced concrete, and composite frame structures (see [12–15]). The state-of-the-art in computational simulation of frame structures subjected to static and dynamic loads is in the non-linear domain to capture the complex behavior of structural systems when approaching their failure range.

Maybe even more important than the simulated non-linear response of a frame structure is its sensitivity to various geometric, mechanical, and material properties defining the structure and to loading parameters. Significant research has been devoted to the general problem of design sensitivity analysis (see [1,2,16,17]). Consistent finite element response sensitivity analysis methods are already well established for displacement-based finite elements (see [4,6,10,18]).

More recently, a procedure for response sensitivity computation using force-based frame elements has been developed [5] by the authors. This new procedure allows the use of force-based frame elements as a powerful simulation tool in applications which require finite element response sensitivity analysis results. Finite element response sensitivities represent an essential ingredient for gradient-based optimization methods needed in structural reliability analysis, structural optimization, structural identification, and finite element model updating (see [7,10]).

This paper presents a careful comparison between the response sensitivity computation methodologies for force-based and displacement-based frame elements in the context of materially-non-linear-only analysis. Both material and discrete loading parameters are considered. Three application examples involving quasi-static and dynamic loadings illustrate the different features of the two formulations in terms of computational effort and accuracy. Consistent finite element response sensitivities are compared with analytical (exact) when available. Conclusions are drawn about the relative merits of the displacement-based and force-based approaches for finite element response sensitivity analysis.

2. Response sensitivity analysis at the structure level

The computation of finite element response sensitivities to material and loading parameters requires extension of the finite element algorithms for response computation only. Let $r(t)$ denote a generic scalar response quantity (displacement, acceleration, local or resultant stress, etc.). By definition, the sensitivity of $r(t)$ with respect to the material or loading parameter θ is expressed mathematically as the (absolute) partial derivative of $r(t)$ with respect to the variable evaluated at $\theta = \theta_0$, i.e., $\partial r(t)/\partial\theta|_{\theta=\theta_0}$ where θ_0 denotes the nominal value taken by the sensitivity parameter θ for the finite element response analysis.

In the sequel, following the notation proposed by Kleiber [10], the scalar response quantity $r(\boldsymbol{\vartheta}) = r(\mathbf{f}(\boldsymbol{\vartheta}), \boldsymbol{\vartheta})$ depends on the parameter vector $\boldsymbol{\vartheta}$ (defined by n time-independent sensitivity parameters, i.e., $\boldsymbol{\vartheta} = [\theta_1, \ldots, \theta_n]^T$), both explicitly and implicitly through the vector function $\mathbf{f}(\boldsymbol{\vartheta})$. It is assumed that $\frac{dr}{d\boldsymbol{\vartheta}}$ denotes the sensitivity gradient or total derivative of r with respect to $\boldsymbol{\vartheta}$, $\frac{dr}{d\theta_i}$ is the absolute partial derivative of the argument r with respect to the scalar variable θ_i, $i = 1, \ldots, n$, (i.e., the derivative of the quantity r with respect to the parameter θ_i considering explicit and implicit dependencies), while $\frac{\partial r}{\partial\theta_i}\big|_{\mathbf{z}}$ is the partial derivative of r with respect to parameter θ_i when the vector of variables $\mathbf{z}$ is kept constant/fixed. In the particular, but important case in which $\mathbf{z} = \mathbf{f}(\boldsymbol{\vartheta})$, the expression $\frac{\partial r}{\partial\theta_i}\big|_{\mathbf{z}}$ reduces to the partial derivative of r considering only the explicit dependency of r on parameter θ_i. For $\boldsymbol{\vartheta} = \theta_1 = \theta$ (single sensitivity parameter case), the adopted notation reduces to the usual elementary calculus notation. The derivations in the sequel consider the case of a single (scalar) sensitivity parameter θ without loss of generality, due to the uncoupled nature of the sensitivity equations with respect to multiple sensitivity parameters.