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Design and sensitivity analysis of dynamical systems subjected to stochastic loading

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Abstract

The paper presents an efficient procedure which allows to carry out reliability-based optimization of linear systems subjected to stochastic loading. The optimization problem is replaced by a sequence of approximate explicit sub-optimization problems that are solved in an efficient manner. Approximation concepts are used to construct high quality approximations of dynamic responses during the optimization process. The approximations are combined with efficient simulation methods to generate explicit approximations of reliability measures in terms of the design variables. The number of dynamic analyses required for the convergence of the design process is reduced dramatically. An efficient sensitivity analysis with respect to the optimization variables and general system parameters becomes possible with the proposed formulation. The sensitivity is evaluated by considering the behavior of the design when the parameters vary within a bounded region. The analysis can identify the degree of robustness of the final design with respect to variations of selected system parameters. A numerical example in terms of a 26-story reinforced concrete building under stochastic earthquake excitation exemplifies the proposed methodology.

Keywords: Approximation concepts; Excursion probability; Linear systems; Optimization; Sensitivity analysis

1. Introduction

The optimization of structural systems subjected to stochastic loading involves the calculation of design parameters to meet some performance criteria with corresponding specified reliability over the service life. The probability that design conditions are satisfied within a particular reference period provides a useful measure of system performance. Such measure corresponds to a first excursion probability problem, which is among the most difficult problems in the area of stochastic dynamics. Early work on the first excursion problem has been focused on analytical and numerical solution methods [1–6]. In general, these methods are limited in application to simple systems of small size. Recently, efficient simulation techniques have been developed to solve first excursion problems of more general systems [7–10]. These methods are robust in application and they have shown promise to be suitable for a wide range of dynamical problems. In an optimization environment, reliability measures have to be evaluated several times before a near optimal design can be obtained. The estimates of the reliability measures at a given design are obtained by carrying out a simulation procedure. Then,

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the evaluation of the reliability measures for every change of the optimization variables requires the evaluation of dynamic responses of the structural system. In general, these responses are nonlinear implicit functions of the optimization variables and they are available only in a numerical way, for instance, by means of a finite element procedure. For systems of practical interest, the repeated evaluation of dynamic responses can be extremely time consuming. Therefore, the use of direct optimization procedures is generally not applicable in the context of structural optimization of systems subjected to stochastic excitation.

In this paper an alternative method to the standard direct optimization method is introduced. A local approximation strategy is used to approximate the system responses as well as the reliability measures. System responses and probability of failures are approximated as functions of the optimization variables during the optimization process. This strategy allows a formal separation of the system reliability analysis from the optimization procedure. The proposed methodology can be seen as a further development of the method presented in Ref. [11]. In the present work, the use of local approximations for the dynamic responses dramatically reduces the number of exact structural analyses required for the convergence of the optimal design process. Also, a very efficient sensitivity analysis with respect to the dependencies of the final design on the variation of system parameters becomes possible. This is due to the availability of the quantities involved in the optimization problem in explicit form. A parameter study can be carried out without any considerable increase in the computational effort. The respective influence on the final design can be determined easily. At the same time, the degree of robustness of the final design with respect to variations of selected system parameters can be obtained directly. In this context, the sensitivity information is crucial to ensure the validity of the final design.

First, the definition of the optimization problem with reliability constraints is presented. The characterization of the system responses as well as the stochastic loading is then considered. Next, approximation concepts and efficient methodologies to estimate excursion probabilities are discussed. Finally, a building structure subjected to a stochastic earthquake excitation is considered as an example problem to show the computational capability and performance of the proposed methodology.

2. Problem definition

The structural optimization problem of systems subjected to stochastic loading can be formulated as the identification of the design vector $\{\theta\}$, θ_i, $i = 1,\ldots,n_d$ such that

Minimize $\quad c(\{\theta\},\{s(\{\theta\})\})$

subject to the design constraints

$$g_q(\{\theta\},\{s(\{\theta\})\}) \leqslant 0, \quad q = 1,\ldots,n_c \;\; \{\theta\} \in \Theta, \tag{1}$$

where $c(\cdot)$ represents the design objective, $\{s(\{\theta\})\}$ denotes the vector of considered reliability measures, n_c is the number of constraints $g_q(\cdot,\cdot)$, and Θ is the set that contains the side constraints for the vector of design variables $\{\theta\}$. In the present formulation, the objective function is assumed to be an explicit function of the design variables only, that is, $c(\{\theta\})$. On the other hand, the probability that some stochastic dynamic responses exceed within a specified time interval $[0,T]$ certain critical upper bounds or fall below critical lower bounds is used as reliability measure. Then, the optimization problem (1) is rewritten as

Minimize $c(\{\theta\})$

subject to the design constraints

$$P_{Fq}(\{\theta\}) = P\left(\bigcup_{i=1}^{n_q}\{\exists t\in[0,T] : r_i^q(t,\{\theta\}) \geqslant \bar{r}_i^q(t) \vee r_i^q(t,\{\theta\}) \leqslant \underline{r}_i^q(t)\}\right) \leqslant P^*_{Fq} \quad q = 1,\ldots,n_c \;\; \{\theta\}\in\Theta, \tag{2}$$

where $r_i^q(t,\{\theta\}),\ i = 1,\ldots,n_q$ are the response functions associated with the failure criterion q evaluated at the design $\{\theta\}$, $\bar{r}_i^q(t)$ is the critical upper bound, $\underline{r}_i^q(t)$ is the critical lower bound, and P^*_{Fq} is the acceptable prescribed or target failure probability for criterion q. It is noted that additional constraints related to general design requirements as well as objective functions dependent on reliability measures can also be included in the optimal design problem. The methodology to be developed in this work can be extended in a straightforward manner for the more general case.

3. Response representation

The general matrix equation of motion for a damped linear structure is given by

$$[M]\{\ddot{x}(t)\} + \{R(\{x(t)\},\{\dot{x}(t)\})\} = [G]\{f(t)\}, \tag{3}$$

where $\{x(t)\}$ is displacement response vector of dimension n, $\{R(\{x(t)\},\{\dot{x}(t)\})\} = [C]\{\dot{x}(t)\} + [K]\{x(t)\}$ is the linear restoring force, $[M]$, $[C]$, and $[K]$ are the mass, damping and stiffness matrices of dimension $n\times n$, $\{f\}$ is the excitation vector of dimension n_f, and $[G]$ is a $n\times n_f$ dimensional matrix that couples the excitation components of the vector $\{f\}$ to the degrees of freedom of the structure. In this paper a modal solution of the dynamic response problem will be used. In the modal solution approach, it is assumed that the dynamic response can be represented by a linear combination of mode shapes: