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Sensitivity analysis for fixed-grid shape optimization by using oblique boundary curve approximation

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Abstract

The remesh-free property is the most attractive feature of the various versions of fixed-grid-based shape optimization methods. When the design boundary curves do not pass through the predetermined analysis grids, however, the element stiffness as well as the stress along the curves may be computed inaccurately. Even with the popular area-fraction-based stiffness evaluation approach, the whole optimization process may become quite inefficient in such a case. As an efficient alternative approach, we considered a stiffness matrix evaluation method based on the boundary curve approximation by piecewise oblique curves which can cross several elements. The main contribution of this work is the analytic derivation of the shape sensitivity for the discretized system by the fixed-grid method. Since the force term in the sensitivity equation is associated only with the elements crossed by the design boundary curve, we only need the design velocities of the intersecting points between the curve and the fixed mesh. The present results obtained for two-dimensional elasticity and Poisson's problems are valid for both the single-scale standard fixed-grid method and the multiscale fictitious domain-based interpolation wavelet-Galerkin method.

Keywords: Sensitivity analysis; Shape optimization; Fixed-grid method; Fictitious domain method; Wavelet-Galerkin method

1. Introduction

In the standard shape optimization based on the finite element approach, remeshing cannot be avoided during the optimization process if accurate analysis is to be guaranteed, especially for design problems

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requiring large shape changes (Bennett and Botkin, 1985; Yao and Choi, 1989). Researchers have shown recent interest on the shape optimization based on the fixed-grid analysis or the Eulerian-type analysis because the analysis offers way to avoid cumbersome remeshing processes. Another advantage of the fixed-grid-based shape optimization method is that it requires only the boundary velocity field for design updates while the standard finite-element-based shape optimization method generally requires design velocities for all nodes, the so-called domain and boundary velocity fields (Choi and Chang, 1994). The fixed-grid based method shows this feature because its analysis nodes are independent of the shape changes.

Though the fixed-grid method is equipped with the excellent remesh-free property, this method has some difficulties in accurately evaluating the stiffness matrices of the elements adjacent to curved boundaries. It has this difficulty mainly because the analysis grids or nodes are always predetermined, and the design boundary does not necessarily pass through these analysis grid points. Since the present sensitivity analysis is mainly for a method to overcome such a difficulty, it is worth stating the implementation technique of the fixed-grid method for shape optimization.

In implementing the remesh-free fixed-grid analysis method, the most popular approach is to embed the original design domain ω encircled by curved boundaries into a fictitious domain Ω usually having a simple geometry. Then, the fixed-grid-based analysis is carried out for Ω. In Fig. 1, we illustrate a rectangular fictitious domain for a generally-shaped ω. The single-scale fixed-grid method usually uses uniformly distributed rectangular finite elements for two-dimensional cases. The stiffness of the elements inside ω is set to be the stiffness of the original material, but the elements inside $\Omega\backslash\omega$ are assigned to have a very weak material. The question is: How does one evaluate the stiffness of the boundary elements lying on the boundary $\partial\omega$. In the fixed-grid method, the stiffness of the boundary elements changes when the boundary curve changes. Therefore, the boundary element stiffness must be estimated accurately for efficient shape optimization.

Until recently, the common approach has been the area-fraction-based stiffness evaluation method, as was used in Garcia and Steven (1998) and Kim and Chang (2003, submitted for publication). The concept of this approach is to evaluate the boundary element stiffness proportionally to the area fraction of the part belonging to ω within the boundary element. The boundary design velocity is thus related to the rate of the change of the area fraction of the boundary element. However, the boundary curve in this method needs to be approximated by zigzags that consist only of vertical and horizontal lines, so this area-fraction-based approach is not effective for curved boundaries. The only way to obtain accurate solutions near the boundary is to work with highly-dense grid distributions.

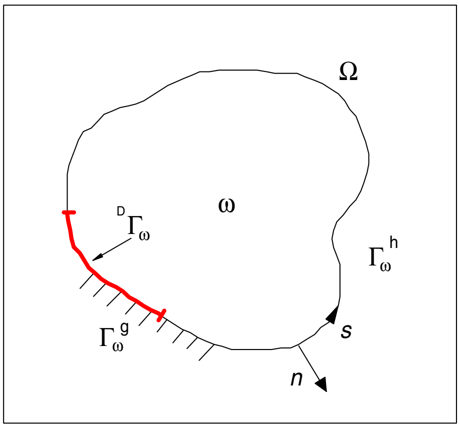

Fig. 1. A two-dimensional problem with the domain of interest ω embedded in a fictitious domain Ω (Γ_ω^g: boundary under kinematic constraint, Γ_ω^h: boundary under natural condition, $^{\mathrm{D}}\Gamma_\omega$: design boundary).