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Optimal operation studies of the power system via sensitivity analysis

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Abstract

This paper presents an optimal operation study of the power system via sensitivity analysis. This study is based on perturbation of optimum theorem, which works with a Lagrangian function associated with the perturbed problem. Starting from an optimal operating point obtained by solution of an optimal power flow problem, the new optimal operating point is calculated directly satisfying the constraints and optimising the objective function after making a small perturbation in the loads. Test results are presented to demonstrate the efficiency of the approach.
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1. Introduction

An important question that often arises in the utilization of mathematical programming algorithms is how much the optimum changes when changes are made in the constraints. An answer to this question is available if the appropriated assumptions are made about conditions holding in the optimum point [1]. This study takes us to general parametric programming problem. The sensibility of the solution for parameter variations, that is, perturbation, has been largely studied since the approach proposed by Dillon [2]. Carpentier et al. [3] applied parametric quadratic programming to the real power economic dispatch problem. Galiana et al. [4] proposed a parametric technique for the optimal power flow (OPF) problem, based on the varying limit strategy. Almeida et al. [5] presented an extension and generalization of the studies described in [4]. Gribik et al. [6] described a parametric OPF formulation to perform sensitivity analysis on incremental losses with respect to the load. The authors determined the sensitivity of the OPF solution to the perturbation in the load, ε, by calculating the perturbed Lagrangian. The sensibility matrix obtained is symmetric and does not contain information on the inequality constraints. Several algorithms have been proposed for sensitivity analysis [7–10]. In this paper, we will make use of the optimal point perturbation theorem, introduced by Fiacco [1]. Applying a small perturbation to an optimal state, it is possible, on the basis of the theorem, to estimate a new optimal state satisfying all the constraints, by optimising the objective function of the problem, and the behavior of the primal and dual system variables can also be studied. While this theorem permits analysis of the effects of perturbations of the objective function and of the inequality and equality constraints, in this study we will only consider the effects of equality constraint perturbations, such as a variation in demand.

The paper is organized as follows: first, the optimal power flow problem is presented, followed by the analysis of optimum perturbation by non-linear programming. Next, an application of this approach is then described. The results

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obtained from tests on three systems (14, 162, 300 buses) are reported. Finally, some concluding remarks are made.

2. Description of the OPF problem

The optimal power flow problem can be formulated as:

$$\begin{aligned} &\text{minimize} && f(x) && \\ &\text{subject to} && g_i(x) = 0 && i = 1, 2, \ldots, m < n \\ & && h_j(x) \le 0 && j = 1, 2, \ldots, p \\ & && x_{\min} \le x \le x_{\max} && \end{aligned} \tag{1}$$

where $x \in R^n$; the control and state variable vector, x, represents the voltage magnitude, angles and LTC's taps. The objective function, $f(x)$, represents the active power losses in the transmission. This function is non-separable and permits no simplifications. The equality constraints, $g(x) = 0$, represent the power flow equations for scheduled load and generation. The inequality constraints, $h(x) \le 0$, represent the functional constraints of the power flow, e.g., limits of active and reactive power flows in the transmission lines and transformers, limits of reactive power injections for reactive control buses and active power injection for the slack bus. This is a typical non-linear and non-convex problem, which is solved by augmented Lagrangian function approach [11].

3. Perturbation of optimum

A perturbation is associated with the equality constraints, $g(x)$, of the problem (1). The inequality constraints, $h(x) \le 0$, represent the functional constraints of the power flow and the state variable vector x. Stated formally, the parametric programming problem is:

$$\begin{aligned} &\min && f(x) && \\ &\text{s.a.} && g_i(x) + \varepsilon_i = 0, && i = 1, \ldots, m \\ & && h_j(x) \le 0, && j = 1, \ldots, r + n \end{aligned} \tag{2}$$

where $\varepsilon = (\varepsilon_i, \ldots, \varepsilon_m)$ is the scalar vector of perturbation.

The Lagrangian function associated with the perturbed problem (2) is:

$$L(x, \mu, \omega) = f(x) + \mu[g(x) + \varepsilon] + \omega h(x) \tag{3}$$

where ω and μ are the Lagrange multipliers.

The necessary condition that a point x^* be a local minimum of problem (2), where f, $\{g_i\}$ and $\{h_j\}$ are twice-differentiable functions, is that there exist vectors μ^* and ω^* such that (x^*, μ^*, ω^*) satisfy

$$\begin{aligned} &\nabla_x L(x, \mu, \omega) = 0 \\ &\omega h(x) = 0 \\ &g(x) = 0 \end{aligned} \tag{4}$$

where $\omega \ge 0$ and μ is unconstrained.

With this assumption, the following set of equations is satisfied at $(x, \mu, \omega) = (x^*, \mu^*, \omega^*)$, $\varepsilon = 0$:

$$\begin{aligned} &\nabla f + \sum \mu(\nabla g_i) + \sum \omega(\nabla h_j) = 0 \\ &\omega_j h_j(x) = 0 \\ &g_i(x) + \varepsilon_i = 0 \end{aligned} \tag{5}$$

The set of non-linear Eq. (5) can be solved by Newton's method at (x^*, μ^*, ω^*), and eliminating the null terms, that gives:

$$\begin{aligned} &\nabla^2_{xx} L(x^*, \mu^*, \lambda^*)\Delta x + \nabla^2_{x\omega} L(x^*, \mu^*, \lambda^*)\Delta\omega \\ &\quad + \nabla^2_{x\mu} L(x^*, \mu^*, \lambda^*)\Delta\mu = 0 \\ &h(x^*)\Delta\omega + \omega^*\nabla h(x^*)\Delta x = 0 \\ &\varepsilon + \nabla g(x^*)\Delta x = 0 \end{aligned} \tag{6}$$

The set of Eq. (6) can be written as follows:

$$\begin{aligned} &\nabla^2_{xx} L(x^*, \mu^*, \lambda^*)\Delta x + \nabla_x h(x^*)\Delta\omega + \nabla_x g(x^*)\Delta\mu = 0 \\ &h(x^*)\Delta\omega + \omega\nabla h(x^*)\Delta x = 0 \\ &\nabla g(x^*)\Delta x = -\varepsilon \end{aligned} \tag{7}$$

The system of Eq. (7) in matrix form is:

$$\begin{bmatrix} \nabla^2_{xx} L(x^*, \omega^*, \mu^*) & \nabla_x h(x^*) & \nabla_x g(x^*) \\ \omega^*\nabla_x h(x^*) & h(x^*) & 0 \\ \nabla_x g(x^*) & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta\omega \\ \Delta\mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \varepsilon \tag{8}$$

To evaluate Δx, $\Delta\mu$ and $\Delta\omega$ from Eq. (8), the optimal solution of the problem, (x^*, μ^*, ω^*), is required. Therefore, the OPF problem must be solved by a method type Lagrangian, where the constraints are associated with Lagrange multipliers (dual variables) as in the method proposed in [11]. The second-order conditions are satisfied for small perturbations ε [1]:

$$\begin{bmatrix} \Delta x \\ \Delta\omega \\ \Delta\mu \end{bmatrix} = \begin{bmatrix} \nabla^2_{xx} L(x^*, \omega^*, \mu^*) & \nabla_x h(x^*) & \nabla_x g(x^*) \\ \omega^*\nabla_x h(x^*) & h(x^*) & 0 \\ \nabla_x g(x^*) & 0 & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \varepsilon \tag{9}$$

where $\Delta x = x(\varepsilon) - x^*$; $\Delta\omega = \omega(\varepsilon) - \omega^*$; $\Delta\mu = \mu(\varepsilon) - \mu^*$.

Therefore, when perturbations are made in the equality constraints, the optimum moves to

$$\begin{aligned} &x(\varepsilon) = \Delta x + x^* \\ &\omega(\varepsilon) = \Delta\omega + \omega^* \\ &\mu(\varepsilon) = \Delta\mu + \mu^* \end{aligned} \tag{10}$$

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