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Design sensitivity analysis and optimization for nonlinear buckling of finite-dimensional elastic conservative structures

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Abstract

The purpose of this review paper is to summarize the existing methods of design sensitivity analysis and optimization of elastic conservative finite-dimensional systems with respect to nonlinear buckling behavior. Difficulties related to geometrical nonlinear singular behaviors are discussed in detail. Characteristics of optimized structures are demonstrated in reference to snapthrough behavior, hill-top branching, and degenerate critical points. A new optimization result of a flexible truss that fully utilizes the snapthrough behavior is also presented.

Keywords: Optimum design; Geometrical nonlinearity; Stability; Sensitivity analysis

1. Introduction

In the early stage of optimum design under buckling constraints, optimal shapes of columns were investigated by analytical approaches. Prager and Taylor [50] derived optimality conditions for columns under linear buckling constraints. Since then, numerous number of works have been published on sensitivity analysis and optimization of column-type structures under linear buckling constraints, where difficulties due to discontinuity of sensitivity coefficients related to multiple eigenvalues have been extensively discussed. Optimization methods of columns under linear buckling constraints are not included in this review article, because they can be found in the literature [14,45,57].

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Optimization of finite-dimensional structures against buckling started in 1970s. Linear buckling formulation was first used neglecting prebuckling deformation. Khot et al. [24] presented an optimality criteria approach for trusses and frames. They applied their method to a shallow truss, although it is clear that prebuckling deformation should be incorporated for those structures. In 1980s, more practical problems were studied incorporating constraints on displacements and stresses as well as linear buckling load factor [33]. Difficulties due to multiple eigenvalues also exist for finite-dimensional structures. Recently, it was shown that the optimum design with multiple linear buckling load factors can be found by successively solving SemiDefinite Programming (SDP) [22,29] without any difficulty by using an interior point method.

Small trusses exhibiting limit point instability were studied in the early stage of optimization of geometrically nonlinear finite-dimensional structures [49]. The maximum total potential energy was also used as the performance measure [23], although it is not clear if maximization of the total potential energy is equivalent to that of the limit point load factor. Kamat and Ruangsingha [21] presented a mathematical programming approach for maximizing limit point loads.

In 1990s, numerical approaches were presented for optimum designs of moderately large geometrically nonlinear structures. Optimality criteria approaches were mainly used for maximizing the limit point load factor [32,55]. Although iterative approaches that are similar to the fully stressed design are simple to implement, the optimality of the solutions derived by those methods is not theoretically clear. Ohsaki and Nakamura [42] presented a method based on parametric programming approach.

For building frames, optimization methods were developed independently from general finite-dimensional structures, because they have unique situation such as brace buckling and interaction of local and global buckling modes [27]. Numerical methods utilizing the characteristics of building frames were developed by Hall et al. [13] and Bažant and Xiang [3]. Hjelmstad and Pezeshk [15] presented an optimality criteria approach for buckling and displacement constraints under lateral loads.

In this paper, methods of sensitivity analysis of geometrically nonlinear buckling loads and formulations of optimization problems are reviewed. Note that problems relating to linear buckling are out of scope of this paper. In the following, *geometrical nonlinearity* means effect of large deformation, where the strains are restricted in a small range. Historical backgrounds as well as scopes for future research are included. Only conservative systems subjected to quasi-static proportional loads are considered. Nonconservative systems, dynamic problems, control problems, and path-dependent problems are beyond scope of this review. Although we concentrate on finite-dimensional systems, the methods and problem formulations presented in this paper are valid also for continuum discretized by a finite element approach.

In Section 2, the basic equations and classification of critical points are briefly presented. In Section 3, possible formulations of optimization problems and difficulties for obtaining optimal solutions are discussed in relation to snapthrough behavior. The existing methods of sensitivity analysis of geometrically nonlinear responses and critical load factors are reviewed in Section 4. The difficulties due to hill-top branching and degenerate critical point are presented in Section 5. In Section 6, existing studies on imperfection sensitivity of optimized structures are shown. Finally, in Section 7, a new result is presented for a flexible truss to generate large deformation efficiently by incorporating snapthrough behavior.

2. Geometrically nonlinear analysis

Consider a finite-dimensional elastic structure subjected to quasi-static proportional loads $\mathbf{P} = \Lambda\mathbf{p}$, where Λ is the load factor and $\mathbf{p}$ is the specified vector of load pattern. Let $\mathbf{A}$ denote the vector of design variables such as stiffnesses of elements and locations of nodes. Note that $\mathbf{p}$ is also a function of $\mathbf{A}$ for the case, e.g., self-weight is considered. The vector of nodal displacements is denoted by $\mathbf{U}(\Lambda,\mathbf{A}) = \{U_i(\Lambda,\mathbf{A})\}$ which is a function of Λ and $\mathbf{A}$. In the following, a subscript is used for indicating an element of a vector.