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Eulerian shape design sensitivity analysis and optimization with a fixed grid

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Abstract

Conventional shape optimization based on the finite element method uses Lagrangian representation in which the finite element mesh moves according to shape change, while modern topology optimization uses Eulerian representation. In this paper, an approach to shape optimization using Eulerian representation such that the mesh distortion problem in the conventional approach can be resolved is proposed. A continuum geometric model is defined on the fixed grid of finite elements. An active set of finite elements that defines the discrete domain is determined using a procedure similar to topology optimization, in which each element has a unique shape density. The shape design parameter that is defined on the geometric model is transformed into the corresponding shape density variation of the boundary elements. Using this transformation, it has been shown that the shape design problem can be treated as a parameter design problem, which is a much easier method than the former. A detailed derivation of how the shape design velocity field can be converted into the shape density variation is presented along with sensitivity calculation. Very efficient sensitivity coefficients are calculated by integrating only those elements that belong to the structural boundary. The accuracy of the sensitivity information is compared with that derived by the finite difference method with excellent agreement. Two design optimization problems are presented to show the feasibility of the proposed design approach.

Keywords: Shape optimization; Topology optimization; Boundary homogenization; Design sensitivity analysis; Fixed grid

1. Introduction

For three decades, remarkable progress has been achieved in geometry-based structural shape optimization [1]. Shape optimization techniques have been successfully integrated with CAD tools, so that design

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variables are chosen from CAD parameters, providing consistency between the design and CAD models [2,3]. A major problem of geometry-based shape optimization is the mesh distortion problem during structural analysis [4]. The regularly distributed mesh at the initial design is often distorted during shape optimization, and as a result, solution accuracy of finite element analysis deteriorates after the initial design. Although many adaptive mesh-regeneration methods have been studied in order to maintain a certain level of solution accuracy, they produce discontinuities in the objective and/or constraints, thus possibly making gradient based optimization complicate [4]. In this paper, conventional shape optimization is referred to as the Lagrangian method since both the geometry and finite element mesh move together during the shape optimization process.

In contrast to the Lagrangian method, a topology optimization method has been developed in order to determine the optimum structural shape without causing any mesh distortion problems [5,6]. The initial geometry of the finite element mesh is maintained throughout the design process, and the material property (shape density) of each element changes as a design variable changes. However, an excessive number of design variables make it difficult to find the optimum design, and results in too many local optimum solutions. In addition, the practicality of the optimum design often raises questions as to the feasibility of manufacturing a structure based on the optimum design. It is non-trivial to determine the structural boundary shape from topology optimization results. In contrast to the shape design, this approach is referred to as the Eulerian method since the shape of the finite element mesh is fixed during the design process.

In this paper, a shape optimization method within the fixed grid framework that uses the efficiency of the adjoint method in the shape sensitivity analysis problem and the advantageous aspects of both conventional shape and topology optimization methods is proposed. The Lagrangian method has the advantage of accurately representing the geometric model, while the Eulerian method relieves mesh distortion problems. During structural analysis, the geometric model is placed over regularly meshed finite elements. The finite elements are fixed during the design process, while the geometric model changes according to the shape design. Finite elements that belong inside the geometric model have a full magnitude of shape density, while those outside the model have a zero magnitude of shape density (a void). Finite elements on the geometric edge have a shape density that is proportional to the area fraction between the material and void. Thus, finite elements on the edge have a shape density between full material and a void. This method is similar to the homogenization method in topology optimization. Thus, in this paper it is referred to as boundary homogenization.

A similar methodology has appeared in the literature. García and Steven [7] applied fixed grid finite element analysis to elasticity problems. Even if the displacement and stress at the boundary oscillate due to the element location to the boundary curve, García and Steven showed that the error reduces as the size of the finite element decreases. They also used the least square method to approximate the stress on the boundary. Using the fixed grid analysis capability, García and Steven also developed design optimization using a fast re-analysis method, which is similar to the discrete semi-analytical method in sensitivity analysis [8,9]. Recently, Woon et al. [10] applied the fixed grid approach to shape optimization using the genetic algorithm, and Kim et al. [11] applied the fixed grid approach to the evolutionary structural optimization (ESO) problem. The ESO process is started by generating a stiffness matrix of the given initial design. Once the matrix is defined, it is solved for displacement and the stress values of each element. ESO then physically removes a small percentage of elements that have low stress values. This completes one cycle of the ESO process. Repeating this process leads to the optimum design. However, the disadvantages of ESO are the expensive solution cost due to the iterative and slow nature of the ESO process. Implementing the fixed grid methodology not only simplifies the mesh generation process, but also allows a very significant reduction of the arithmetic calculations necessary to update the stiffness matrix for the modified topology, instead of a full regeneration of the matrix.

In shape optimization, a shape change in the geometric model produces a shape density change in the finite elements on the edge. As the structural shape changes, a new shape density is calculated for the ele-