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Sensitivity Analysis for Parametric Completely Generalized Strongly Nonlinear Implicit Quasi-Variational Inclusions

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Abstract—In this paper, by using a resolvent operator technique of maximal monotone mappings and the property of a fixed-point set of set-valued contractive mappings, we study the behavior and sensitivity of the solutions of the parametric completely generalized strongly nonlinear implicit quasi-variational inclusion in Hilbert space. Our results extend, improve, and unify the previously many results in this field. © 2005 Elsevier Ltd. All rights reserved.

Keywords—Parametric completely generalized strongly nonlinear implicit quasi-variational inclusions, Sensitivity analysis, Resolvent operator, Hilbert space.

1. INTRODUCTION

It is well known that variational inequality theory and complementarity problem theory play an important and fundamental role in the study of a wide class of problems arising in differential equations, mechanics, physics, optimization and control, nonlinear programming, economics and transportation equilibrium, and engineering sciences, etc. (see [1–12]). A useful and important generalization of variational inequality is called the quasi-variational inclusion. Ding [13], Huang [14], and Noor [15] have used the resolvent operator techniques of maximal monotone mappings to study the existence of quasi-variational inclusions.

Sensitivity analysis of a solution set for variational inequalities has been studied by many authors. Dafermos [16], Mukherjee and Berma [17], Noor [18], and Yen [19] used the projection technique to deal with the sensitivity analysis of solutions for variational inequalities with single-valued mappings. Robinson [20] used the implicit function approach with normal mappings and studied the sensitivity analysis of solutions for variational inequalities in finite-dimensional spaces. By using resolvent operator technique, Adly [21], Noor and Noor [22], Agrawal, Cho and Huang [23], and Noor [24] study the sensitivity analysis of solutions for the quasi-variational

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inclusions with single-valued mappings; Ding [25] studied the behavior and sensitivity analysis of solutions for generalized nonlinear implicit quasi-variational inclusions; Liu, Debnath, Kang and Ume [26] studied the behavior and sensitivity analysis of solutions for parametric completely generalized nonlinear implicit quasi-variational inclusions.

Inspired and motivated by recent research works in this field, in this paper, by using implicit resolvent operator technique and the property of fixed-point set of set-valued contractive mappings, we study the behavior and sensitivity analysis of solutions of a new class of parametric completely generalized strongly nonlinear implicit quasi-variational inclusions with multivalued and single-valued nonlinear mappings in Hilbert space. Our results extend, improve, and unify the corresponding results in [23–26] and the references therein.

2. PRELIMINARIES AND DEFINITIONS

Let H be a real Hilbert space with norm and inner product denoted by $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$, respectively. Let $C(H)$ denote the families of all nonempty compact subsets of H, and $\tilde{H}(\cdot,\cdot)$ denote the Hausdorff metric on $C(H)$ defined by

$$\tilde{H}(A,B)=\max\left\{\sup_{a\in A}d(a,B),\sup_{b\in B}d(A,b)\right\},\qquad \forall A,B\in C(H),$$

where $d(a,B)=\inf_{b\in B}\|a-b\|$, $d(A,b)=\inf_{a\in A}\|a-b\|$.

Let Γ be a nonempty open subset of H in which the parameter λ takes values, $N,M: H\times H\times\Gamma\longrightarrow H$, $m,g: H\times\Gamma\longrightarrow H$ be single-valued mappings, and $A,B,C,D,E,F,G: H\times\Gamma\longrightarrow C(H)$ be multivalued mappings. Let $W: H\times H\times\Gamma\longrightarrow 2^H$ be a set-valued mapping such that for each given $(z,\lambda)\in H\times\Gamma$, $W(\cdot,z,\lambda): H\longrightarrow 2^H$ is a maximal monotone mapping with $(E(H,\lambda)-m(H,\lambda))\cap\operatorname{dom}W(\cdot,z,\lambda)\neq\emptyset$. Throughout this paper, unless otherwise stated, we will consider the following parametric completely generalized strongly nonlinear implicit quasi-variational inclusion problem (in short, PCGSNIQVIP):

$$\begin{array}{l}\text{for each fixed } \lambda\in\Gamma \text{ and } f\in H,\text{ find } x(\lambda)\in H,\ u(\lambda)\in A(x(\lambda),\lambda),\\ v(\lambda)\in B(x(\lambda),\lambda),\ w(\lambda)\in C(x(\lambda),\lambda),\ p(\lambda)\in D(x(\lambda),\lambda),\ s(\lambda)\in\\ E(x(\lambda),\lambda),\ z(\lambda)\in F(x(\lambda),\lambda),\ n(\lambda)\in G(x(\lambda),\lambda),\text{ such that } f\in\\ N(u(\lambda),v(\lambda),\lambda)-M(w(\lambda),p(\lambda),\lambda)+W(s(\lambda)-m(n(\lambda),\lambda),z(\lambda),\lambda).\end{array}\tag{2.1}$$

Special Cases

(I) If $E=g: H\times\Gamma\longrightarrow H$ is a single-valued mapping and $G(x,\lambda)=x$ for all $(x,\lambda)\in H\times\Gamma$, then the problem (PCGSNIQVIP) (2.1) is equivalent to the following parametric completely generalized nonlinear implicit quasi-variational inclusion problem:

$$\begin{array}{l}\text{for each fixed } \lambda\in\Gamma \text{ and } f\in H,\text{ find } x(\lambda)\in H,\ u(\lambda)\in A(x(\lambda),\lambda),\\ v(\lambda)\in B(x(\lambda),\lambda),\ w(\lambda)\in C(x(\lambda),\lambda),\ p(\lambda)\in D(x(\lambda),\lambda),\ z(\lambda)\in\\ F(x(\lambda),\lambda),\text{ such that } (g-m)(x(\lambda),\lambda)-m(x(\lambda),\lambda))\in\operatorname{dom}W(\cdot,z,\lambda).\\ f\in N(u(\lambda),v(\lambda),\lambda)-M(w(\lambda),p(\lambda),\lambda)+W((g-m)(x(\lambda),\lambda),z(\lambda),\lambda).\end{array}\tag{2.2}$$

Problem (2.2) was introduced and studied by Liu, Debnath, Kang and Ume [26].

If $f=0$, $M(x,y,\lambda)=0$ for all $(x,y,\lambda)\in H\times H\times\Gamma$, $m(x,\lambda)=0$ and $A(x,\lambda)=B(x,\lambda)=F(x,\lambda)=x$ for all $(x,\lambda)\in H\times\Gamma$, then problem (2.2) is equivalent to the following parametric generalized quasi-variational inclusion problems.

For each fixed $\lambda\in\Gamma$, find $x(\lambda)\in H$, such that $g(x(\lambda),\lambda)\in\operatorname{dom}W(\cdot,z,\lambda)$.

$$0\in N(x(\lambda),x(\lambda),\lambda)+W(g(x(\lambda),\lambda),x(\lambda),\lambda).\tag{2.3}$$

Problem (2.3) was introduced and studied by Noor [24].