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Adaptivity in linear elastic fracture mechanics based on shape sensitivity analysis

Roberto Saliba [a], Claudio Padra [a], Marcelo J. Vénere [b], Edgardo Taroco [c], Raúl A. Feijóo [c,*]

[a] *Centro Atómico Bariloche, 8400 Bariloche, Argentina*

[b] *PLADEMA–CNEA, Facultad de Exactas, Universidad Nacional del Centro de la Provincia de Buenos Aires, Pinto 399, 7000 Tandil, Argentina*

[c] *LNCC/CNPq, Laboratório Nacional de Computação Científica, Getulio Vargas 333, Quitandinhas, 25651-075 Petrópolis, RJ, Brasil*

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Abstract

If crack growth of an elastic body is viewed as a shape change we can use the well known concept of shape sensitivity analysis to compute the energy release rate. To do this, we adopt as cost function the total potential energy and as state equation the equilibrium equation. The shape derivative of the total potential energy stored in the cracked body $\dot{\Pi}$ depends on the displacement field $\mathbf{u}$ and on the shape change velocity field V which characterize the crack growth. Following this procedure the present paper deals with the derivation of a novel a posteriori error estimator which is an upper bound of the global error $|\dot{\Pi} - \dot{\Pi}_h|$. This error estimator has been specifically designed to evaluate the energy release rate in mesh refinement or re-meshing procedures so as to obtain improved meshes for which the optimal rate of convergence is recovered even in a case of singularities. This novel estimator is capable to capture all source of errors for the energy release rate $\dot{\Pi}$: the ones from stress concentration and the errors from the sensitivity of the solution to shape changes due to crack growth. Finally, well known three-dimensional examples of un-cracked and cracked body are considered in order to illustrate the potentiality of the proposed methodology.

Keywords: Fracture mechanics; Sensitivity analysis; A posteriori error estimation; Adaptive analysis; Finite element method

* Corresponding author.

E-mail addresses: salibar@cab.cnea.gov.ar (R. Saliba), padra@cab.cnea.gov.ar (C. Padra), venerem@exa.unicen.edu.ar (M.J. Vénere), etam@lncc.br (E. Taroco), feij@lncc.br (R.A. Feijóo).

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1. Introduction

An upstanding concept in linear elastic fracture mechanics is the total potential energy release rate. This concept was introduced by Griffiths [6] applying the energy conservation principle in the analysis of cracked bodies. As known, the energy release rate means the rate of change with respect to the *crack advance* of the energy available for fracture. In other words, the derivative of the total potential energy with respect to the crack growth.

Due to the difficulty to obtain expressions for the potential energy as an explicit function of the crack length, which enable us to obtain derivatives in a direct form, several numerical and experimental procedures, have been developed in fracture mechanics.

However, the Shape Sensitivity Analysis (proposed originally by Cèa [4], Zolésio [20], and widely discussed by Haug et al. [8]) has been successfully applied as a systematic methodology to obtain the expression of the potential energy release rate in cracked bodies. As stated in [10], the crack growth can be interpreted as a *shape change* of the body. Then, using well known results of Shape Sensitivity Analysis, the general expression for the energy release rate as an integral over the initial configuration of a cracked body was obtained. Moreover, this expression is a function of a velocity field describing the change of the initial shape and which simulates the crack advance. Further, using the Finite Element Method and post processing techniques, several applications in three-dimensional linear elastic cracked bodies were presented in [10] showing the potentiality of this approach.

Following this technique, one question arise: *is it possible to find an* a posteriori *error estimator which can be applied in an adaptive finite element energy release rate analysis?* Before answering this question, some considerations are in order. The adaptive method for linear elasticity equations, which is based on the standard a posteriori error estimator for the total potential energy, ignore the particular velocity field used to simulate the crack advance. However, for each velocity field, an adequate adaptive procedure might be able to find the best sequence of meshes to compute sharply the energy release rate.

The present paper deals with the derivation of an a posteriori error estimator for shape sensitivity analysis, which has been specifically designed to evaluate the energy release rate in an adaptive finite element scheme. This estimator can be used to adapt the mesh automatically so as to increase the accuracy of the energy release rate finite element computations. To this end, and using the well known terminology of shape optimization, we adopt as a cost function the total potential energy of an isotropic linear elastic cracked body under infinitesimal deformation and, as a restriction, the variational formulation (weak form) of the static equilibrium. We also use the so-called (continuous) *direct method* for the evaluation of the sensitivity of the total potential energy with respect to the shape change of the cracked body under consideration.

The outline of the paper is as follows. In Section 2 we recall the formulation of the elasticity problem and the total potential energy properly posed to apply the shape sensitivity analysis. In Section 3, we shall briefly describe the shape sensitivity analysis (and its finite element approximations) for both the displacement and the total potential energy by mean of a velocity field. We shall use this velocity field to simulate the changes in the shape due to the crack growth. In Section 4 we will introduce the a posteriori error estimator and will prove that this estimator provide upper estimates for the energy release rate. In order to show the potentiality of the present approach, several numerical experiments are presented in Section 5 where the characterization of the velocity field and the adaptive scheme are also discussed (Sections 5.1 and 5.2, respectively). Finally, Section 6 will be devoted to conclusions.

2. Statement of the problem

In the present section we introduce the state equation and the cost function, using the well know terminology of shape optimization. We will briefly review equations which governs the infinitesimal deformations of an isotropic linear elastic solid under the action of external forces.