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From Experiments to Hydrodynamic Traffic Flow Models: II—Sensitivity Analysis

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Abstract—This paper deals with a parameter sensitivity analysis for a class of traffic flow macroscopic models obtained by mass conservation equation linked to a phenomenological model suitable to relate the local mass velocity to the density of vehicles. The analysis allows to focus how the result of simulations is modified by variations of the parameters. A suitable functional is introduced to give a measure of this sensitivity. © 2005 Elsevier Ltd. All rights reserved.

Keywords—Traffic flow, Hydrodynamics, Conservation equations, Parameter identification.

1. INTRODUCTION

This paper develops a parameter sensitivity analysis for a class of first-order hydrodynamical models of vehicular traffic flow, which is obtained by mass conservation equation properly closed by a phenomenological relation linking the local mass velocity to density and density gradients.

Indeed, it is a simple class of models however useful for the applications as documented in recent papers [1,2] dealing, respectively, with roads with variable geometrical conditions and with large road networks. In fact, the computational complexity induced by the analysis of networks requires simple models to describe the flow conditions in the links. Relatively more complex models are obtained by using linear momentum equations rather than the above-mentioned relation between velocity and density. The interested reader may be referred to the recent literature [3–5], for deeper information on the above topic.

The specific class of models we deal with in this paper was first proposed in [6] and then developed in [7]. The identification of the parameters of the model has been developed by different methods in [8,9]. The qualitative analysis of the solutions can be recovered in [10].

The parameters identification proposed in [9] has shown that each parameter spans a range of values. This means that different flow conditions, related to different hours, weather conditions, and so on, correspond to different quantitative descriptions of the models, although their qualitative behavior remains the same. As a consequence of these parameter fluctuations a sensitivity

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analysis is useful to test how their variation may affect the solutions. To develop the above analysis different flow conditions are considered, which technically means that mathematical problems with different initial-boundary conditions are simulated. The influence of the parameters over the above simulations is analyzed by the introduction of a suitable functional that allows us to compute the distance between different results, due to the variation of the parameters.

The paper is proposed through three more sections: Section 2 deals with the description of the mathematical model and with the parameter identification developed in [9]. Section 3 deals with the parameter sensitivity analysis. The final section proposes a critical analysis on the contents of this paper.

2. MATHEMATICAL FRAMEWORK AND IDENTIFICATION OF PARAMETERS

The flow of vehicles is assumed one directional along a one lane road with length l. It is convenient introducing dimensionless variables normalized in order to take values in the interval $[0,1]$. The characteristic time T and the length l, as well as maximum density n_M and maximum average velocity v_M must be introduced. Specifically: n_M is the maximum density of vehicles corresponding to bump to bump vehicle distance; v_M is the maximum mean velocity which may be reached by vehicles in free flow; T is the time necessary to cover the whole road of length l at the maximum mean velocity. Using the above quantities the following independent variables are identified:

- $t = t_r/T$ is the dimensionless time variable referred to T (t_r is the real time);
- $x = x_r/l$ is the dimensionless space variable referred to the length of the road l (x_r is the real one-dimensional space).

Two dimensionless dependent variables are considered:

- $u = n/n_M$ is the dimensionless density referred to the maximum density n_M of vehicles;
- $v = v_r/v_M$ is the dimensionless velocity referred to the maximum mean velocity v_M (v_r is the real velocity).

Following Sections 4 and 5 of [4], the model is obtained by the mass conservation equation,

$$\frac{\partial u}{\partial t} + \frac{\partial (uv)}{\partial x} = 0, \tag{2.1}$$

properly closed by a phenomenological model linking v to u and $\frac{\partial u}{\partial x}$.

In paper [9] a specific phenomenological model is introduced that is based on field data collected on an Italian motorway.

At first the velocity-density relation at equilibrium is considered; on the basis of the experimental data it seems to be described by the following analytical model:

$$v_e(u) = \exp\left\{-\alpha \frac{u}{1-u}\right\}, \tag{2.2}$$

where α is a positive parameter spanning in the interval $[1, 2.5]$.

In steady nonuniform conditions, that is when the flow q is constant along the x-axis, the density may change with the x variable. Following the line of papers [6,7] the apparent density u^* is considered

$$u* = u\left[1 + \eta(1-u)\frac{\partial u}{\partial x}\right], \qquad \eta > 0, \tag{2.3}$$

and replaced in equation (2.2). Then the following phenomenological model is obtained:

$$v = v\left(u, \frac{\partial u}{\partial x}; \alpha, \eta\right) = \exp\left\{-\alpha \frac{u\left[1 + \eta(1-u)\frac{\partial u}{\partial x}\right]}{1 - u\left[1 + \eta(1-u)\frac{\partial u}{\partial x}\right]}\right\}, \tag{2.4}$$

where the parameter η spans in the interval $[0.1, 0.5]$.