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# First-order differential sensitivity analysis of a nuclear safety system by Monte Carlo simulation

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**Abstract**

In this paper we employ a Monte Carlo method to compute the first-order, differential sensitivity indexes of the basic events characterizing the reliability behavior of the containment spray injection system of a nuclear power plant. An exemplification is provided as to how the obtained sensitivity indexes can be used to drive improvements in the system design and operation.


*Keywords:* Monte Carlo simulation; Sensitivity analysis; Containment spray injection system; Nuclear safety system

## 1. Introduction

In the nuclear field one of the principal activities for applications of risk-informed regulatory processes is the ranking of structures, systems and components with respect to their safety-significance [1,2]. With reference to safety systems in particular, this requires an analysis of how the safety performance is affected by the stochastic behavior of the components constituting the system: importance measures [1–3] and sensitivity indexes [4] are often used for this scope [3]. When realistic issues of system operation are included, such as components' ageing and maintenance, load-sharing, etc. and when uncertainty in the components parameters' values exist, the computation of these measures and indexes is not straightforward [5].

In this paper we illustrate a Monte Carlo simulation method which allows to compute first-order, differential sensitivity indexes [6–8]. The details of the method are illustrated and a realistic case of a nuclear safety system, the Containment Spray Injection System (CSIS) [7] is presented. The sensitivity indexes obtained are used to suggest changes in the system design and operation.

In the application presented, the Monte Carlo calculations are standard analog ones (no biasing). The introduction of biasing techniques can be easily accommodated in the method to reduce the variance of the Monte Carlo estimates. The calculations have been performed by means of the MARA (Monte Carlo Availability Reliability Analysis) code, developed by the authors at the Department of Nuclear Engineering of the Polytechnic of Milan.

In Section 2, we present the Monte Carlo method for the computation of first-order, differential sensitivity indexes. Section 3 contains the results of the application of the method to the Containment Spray Injection System. In Section 4, the findings are summarized and some remarks are given with respect to the method proposed.

## 2. Monte Carlo simulation of system transport

The stochastic transport of the states of an engineered system, within a reliability and availability analysis is best described by a non-linear integral transport equation in the dependent variable $\psi(\tau,k)$ defined below, which can take into account the various phenomena (single and dependent failures, repair, ageing, maintenance, etc.) which affect the system life analysis [6,10]. In practice, since the transport equation for $\psi(\tau,k)$ still lacks of an explicit general analytic solution, the Monte Carlo simulation method seems to be the only viable approach suitable for assessing the functionals of interest under practical, realistic conditions.

At a given time $t$, the system state, i.e. the configuration of its $N_C$ components, is represented by a point $(k,t)$ in

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**Nomenclature**

| | |
|---|---|
| CLCS | consequence limiting control system |
| CSIS | containment spray injection system |
| LOCA | loss of coolant accident |
| LWR | light water reactor |
| RWST | refuelling water storage tank |

the system phase space, where $k \in Z$ is an integer index which codes all the possible system configurations $(j_1, j_2, \ldots, j_{N_C})$, with $j_i$ being an integer which codes the state of the $i$th component. The functionals to be estimated are of the kind:

$$G(t) = \sum_{k \in \Gamma} \int_0^t \psi(\tau, k) R_k(\tau, t) \mathrm{d}\tau \qquad t \in [0, T_\mathrm{M}] \tag{1}$$

where $\psi(\tau,k)$ is the ingoing collision density, i.e. the probability density of entering state $k$ at time $\tau$, $\Gamma$ is the set of possible system states which contribute to the function of interest $R_k(\tau,t)$ and $T_\mathrm{M}$ is the mission time. For a fixed time $t \in [0,T_\mathrm{M}]$, the quantity defined by Eq. (1) has the general form of an expected value of the kind

$$G(t) = \int f(x) g(x, t) \mathrm{d}x \qquad t \in [0, T_\mathrm{M}] \tag{2}$$

where $g(x,t) = R_k(\tau,t)$ is a function of the vector of stochastic variables $x \equiv (\tau, k)$ distributed according to $f(x) = \psi(\tau,k)$ which is a probability density function with respect to the continuous variable $\tau$ and a probability mass function with respect to the discrete variable $k$.

The functionals we are interested in are the system unreliability $U_\mathrm{R}(t)$ and unavailability $U_\mathrm{A}(t)$ at time $t$, so that $\Gamma$ is the subset of all the system failed states and $R_k(\tau,t)$ is unity, in the former case of unreliability, or the probability of the system not exiting before $t$ from the failed state $k$ entered at $\tau < t$, in the latter case of unavailability [6]. Note that the above expression (1) is quite general, independent of any particular system model which generates the $\psi(\tau,k)$s.

For the generic time $t \in [0,T_\mathrm{M}]$, the quantity $G(t)$ in Eqs. (1) and (2) depends on a vector of parameters $p$ (e.g. in our case of interest the components failure and repair rates, the maintenance intervals, etc.) which appear in the function $g(x,t,p) = R_k(\tau,t,p)$ and in $f(x,p) = \psi(\tau,k,p)$. For simplicity of notation we re-write (2) by expliciting the dependence on the parameters $p$ and neglecting the dependence on the time $t$

$$G(p) = \int g(x, p) f(x, p) \mathrm{d}x \tag{3}$$

We now refer to the case of $G$ depending on only a single parameter $p$ and describe a procedure for the estimate of the first-order sensitivity of $G$ with respect to a variation of $p$, namely $\mathrm{d}G/\mathrm{d}p$ [6].

Let us set, for brevity

$$g \equiv g(x, p); \quad g^* \equiv g(x, p + \Delta p)$$

$$f \equiv f(x, p); \quad f^* \equiv f(x, p + \Delta p) \tag{4}$$

Further, let us indicate with $E[\cdot]$ and $E^*[\cdot]$ the expected values of the argument calculated with the pdfs $f$ and $f^*$, respectively.

Corresponding to the value $p + \Delta p$ of the parameter, the definite integral (3) becomes

$$G^* \equiv G(p + \Delta p) = \int g(x, p + \Delta p) \cdot f(x, p + \Delta p) dx = E^*[g^*] \tag{5}$$

But we also have

$$G^* \equiv G(p + \Delta p) = \int g(x, p + \Delta p) \frac{f(x, p + \Delta p)}{f(x, p)} f(x, p) \mathrm{d}x$$
$$= E\left[g^* \frac{f^*}{f}\right] \equiv E[h] \tag{6}$$

where we have set

$$h(x, p, \Delta p) = g(x, p + \Delta p) \frac{f(x, p + \Delta p)}{f(x, p)} \equiv g^* \frac{f^*}{f} \tag{7}$$

Corresponding to a given $\Delta p$ (in general such that $\Delta p / p \ll 1$), the Monte Carlo estimate of $G^*$ can be done simultaneously to that of $G$: for each of $N$ values $x_i$ sampled from the pdf $f(x,p)$, we accumulate the realization $g(x_i,p)$, with the aim of calculating the sample mean $G_N = \bar{g}$ as an estimate of $G$, and we also accumulate the realization $h(x_i,p,\Delta p)$ with the aim of calculating the sample mean $G_N^* = \bar{h}$ as an estimate of $G^*$. By so doing, the sample means $G_N$ and $G_N^*$, calculated by using the same sample $\{x_i\}$ are obviously correlated.

To compute the sensitivity of $G$ with respect to the variation of the parameter from $p$ to $p + \Delta p$, let us define

$$\Delta G_N = G_N^* - G_N = \frac{1}{N} \sum_{i=1}^{N} (h_i - g_i) = \bar{h} - \bar{g} \tag{8}$$

where for brevity, we set

$$h_i \equiv h(x_i, p, \Delta p), \quad g_i \equiv g(x_i, p) \tag{9}$$

The sensitivity $\partial G / \partial p$ is estimated as:

$$E\left[\frac{\Delta G_N}{\Delta p}\right] = \frac{1}{\Delta p} E[h_i - g_i] = \frac{1}{\Delta p} (G^* - G) \approx \frac{1}{\Delta p} (\bar{h} - \bar{g}) \tag{10}$$