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A general approach to sensitivity analysis of fluid–structure interactions

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Abstract

This paper presents a general monolithic formulation for sensitivity analysis of the steady-state interaction of a viscous incompressible flow with an elastic structure undergoing large displacements (geometric nonlinearities). The problem is solved in a direct implicit manner using a Newton–Raphson adaptive finite element method. A pseudo-solid formulation is used to manage the deformations of the fluid domain. The formulation uses fluid velocity, pressure, and pseudo-solid displacements as unknowns in the flow domain and displacements in the structural components. The adaptive formulation is verified on a problem with a closed-form solution. It is then applied to sensitivity analysis of an elastic cylinder placed in a uniform flow. Sensitivities are used for fast evaluation of nearby problems (i.e. for nearby values of the parameters) and for cascading uncertainty through the Computational Fluid Dynamics/Computational Structural Dynamics code to yield uncertainty estimates of the cylinder shape.

Keywords: Fluid; Structure; Interactions; Monolithic; Sensitivity

1. Introduction

Interaction between solids and fluids has been a topic of engineering interest for many years. The behavior of vessels subject to wave loads, of planes in flight as well as that of submarines or transmission lines in the wind are but a few examples.

This paper presents a formulation suitable for simulating the interaction between an incompressible flow and a structure undergoing large displacements and for computing its sensitivities with respect to parameters of interest. We assume existence and uniqueness of the solution. The interested reader is referred to Dowell and Hall (2001) for a review of fluid–structure interaction (FSI) and Grandmont (2002), Desjardins and Esteban (2000), for mathematical discussion of existence and uniqueness. Previous works on sensitivity analysis of FSI are by Lund et al. (2001, 2003), Moller and Lund (2000), Ghattas and Li (1998) and Fernandez and Moubachir (2001).

In many instances, interaction between fluids and solids is achieved through weak or loose coupling of specialized softwares. This is very cost-effective because it requires little changes to analysis modules and takes advantage of expertise of each discipline. Hydrodynamic loads obtained by Computational Fluid Dynamics are transferred to the structural model to predict solid displacements which are then transferred back to the fluid module until convergence to

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reflect changes in the geometry. Because data transfers are approximate, equilibrium at the interface between solid and fluid is not perfectly satisfied. Such departures from equilibrium are often magnified to unacceptable levels if the approach is used in an optimization loop. Sensitivities at the interface may become erroneous. We introduce an implicit fully coupled (or direct implicit monolithic) formulation that avoids this problem. A pseudo-solid formulation is introduced to manage in an implicit manner the deformations of the fluid domain. An implicit treatment of the interface equilibrium achieves monolithic coupling of the fluid and solid problems. This approach delivers quadratic convergence of Newton's method and ensures quality solutions field for the sensitivity analysis.

To cope with the deformations of both boundaries and fluid mesh, several techniques have been studied and discussed in the literature for unstructured meshes. They can be classified in three categories. Firstly, one can remesh the fluid domain completely, once the deformation of the structure is known, and iterate until convergence (Heil, 1998). Secondly, one can use a spring analogy (linear and torsional) to move the grid points (Farhat et al., 1998; Degand and Farhat, 2001) or a Laplacian (Moller and Lund, 2000; Fernandez and Moubachir, 2005). Finally, the pseudo-solid approach introduces structural-like equations to manage the deformation of the fluid domain at the continuum level (Sackinger et al., 1996). Such pseudo-solid material laws have been used in various forms (Lohner and Yang, 1996; Lund et al., 2001, 2003; Chiandussi et al., 2000; Oñate and Garcia, 2001; Nielsen and Anderson, 2001). We have opted for the pseudo-solid approach of Sackinger et al. (1996) formulated at the continuum level, because it allows for a full coupling of all components. We go further by solving the fluid, structural and pseudo-solid equations in an implicit fully coupled manner, which results in an *implicit monolithic* method. Such a formulation is also compatible with our general approach for error estimation and mesh adaptation procedures. Finally, using this approach, sensitivities are straightforward to obtain and implement by the sensitivity equation method (SEM).

Approaches to sensitivity computations differ depending on the order of the operations of discretization and differentiation. In the discrete sensitivities approach, the total derivative of the computational model with respect to the parameter is calculated (Haug et al., 1986), whereas in the continuous sensitivity equation (CSE) method one differentiates the continuum equations to yield differential equations for the sensitivities (Borggaard and Burns, 1997). Note that sensitivity analysis is more advanced in structural mechanics than in fluid mechanics (Haug et al., 1986; Borggaard and Burns, 1997). Furthermore, there is a paucity of literature on sensitivity analysis of fluid–structure interaction. Ghattas and Li (1998) use the discrete SEM and exploit the Newton-like method developed for the fluid–structure equations to solve the fluid–structure sensitivity equations. They iterate between the solid and the fluid meshes, as terms coupling the solid displacements to the fluid have been neglected. These terms would naturally arise from the coupling between pseudo-solid/fluid and solid/pseudo-solid. Since Ghattas and Li (1998) do not use a pseudo-solid approach, the coupling of tractions at the interface cannot be treated implicitly. Thus, the sensitivity solution can only be obtained iteratively. The work of Lund et al. (2001) uses an iterative hybrid approach in which part of the sensitivity formulation is discrete while the other is continuous. Additional simplifications are also made to the global system. This hybrid approach has been applied to the optimal design of flexible structures undergoing large displacements induced by the flow (Lund et al., 2001, 2003). The present work avoids this approximation through the monolithic coupling approach to the fluid–structure problem. In this paper, we present a fully CSE method derived from the pseudo-solid formulation of the fluid–structure interaction problem. The sensitivity problem shares many similarities with the fluid–structure problem.

The paper begins with the description of the steady-state governing equations for laminar incompressible fluids, hyperelastic solid behavior, pseudo-solid material law, fluid–structure interface equilibrium. Then sensitivity equations are derived. The weak forms of the equations are then presented. The next sections detail the solution procedure for coupling the fluid and solid domains, the adaptive finite element procedure for the fluid–structure interaction problem and some specific aspects related to mesh management. The methodology is verified on a problem with a closed-form solution. It is then applied to sensitivity and uncertainty analysis for a flexible cylinder in a uniform flow. We also illustrate the use of sensitivity information for fast evaluation of nearby problems (i.e. for nearby values of the parameters) and for uncertainty analysis. The paper ends with conclusions.

2. Governing equations

The steady flow of an incompressible fluid is described by the continuity and momentum equations (Schlichting, 1979),

$$\nabla \cdot \mathbf{u}_f = 0, \tag{1}$$

$$\rho_f \mathbf{u}_f \cdot \nabla \mathbf{u}_f = \nabla \cdot \boldsymbol{\sigma}_f + \mathbf{f}_f, \tag{2}$$