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Comput. Methods Appl. Mech. Engrg. 195 (2006) 462–478
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

A new adaptive remeshing scheme based on the sensitivity analysis of the SPR point wise error estimation

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Received 7 January 2004; received in revised form 7 October 2004; accepted 30 October 2004

Abstract

This paper presents a formulation for the obtainment of the sensitivity analysis of a point wise error estimator with respect to the nodal coordinates using the adjoint state method. The proposed point wise error estimator is based on the SPR method.

The numerical accuracy of the presented sensitivity analysis has been tested by perturbing a mesh. The capability of the presented sensitivity analysis for the detection of pollution error has also been tested.

A new adaptive remeshing strategy based on the sensitivity analysis of the point wise error estimation has been developed and tested. This strategy produces very cheap meshes for the accurate evaluation of stresses at specific points.

Keywords: Error estimation; Sensitivity analysis; Adaptivity; Point wise error; Pollution error

1. Introduction

In the solid mechanics context, the idea of improving the mesh quality by using a sensitivity analysis is not new. The first historical reference comes from McNeic and Marcal [1], who proposed to improve a mesh by minimizing the potential energy of the structural problem with respect to the nodal coordinates. This was done by coupling the finite element solution with the mesh sensitivity analysis. Another important reference comes from Kang, Tae and Kwak [2], who maximized the deformation energy with respect to the nodal coordinates. These works allow to optimize a mesh in order to obtain the best solution for a

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0045-7825/$ - see front matter
doi:10.1016/j.cma.2004.10.015

structural problem with a fixed mesh topology and a fixed number of degrees of freedom. These types of methods are very convenient if the storing capacity and the computational speed are limited. Nevertheless, the increasing power of computers makes the computational power not to be a serious limitation. Due to that, this type of methods is no more in use.

On the other hand, it is possible to obtain specific adaptive remeshing strategies for the control of the point wise error of any physical quantity related with the structural analysis (see Bugeda [3]), but the control of the pollution error (see Babuška [4,5]) obligates to control the local error everywhere, this producing globally adapted meshes with a very high number of degrees of freedom. Nevertheless, when doing a structural analysis we know very often where the points with the highest values of stresses are, and the control of the point wise error not only at those points but everywhere seems to be excessively expensive for just evaluating the stresses at those points.

Furthermore, it is well known (see Babuška [4,5]) that a local refinement around the conflictive points will only minimise the so called "local error", but not the "pollution error". For this reason, the evaluation of the sensitivity analysis of the point wise error at those points with respect to the nodal coordinates can provide information about which zones of a mesh have the biggest influence over the point wise error in stresses at those points. This information indicates how to control that error in the most effective way by refining the mesh only at those zones.

Duality techniques have already been applied in order to get element bounds for functional outputs of different partial differential equations and the corresponding goal-oriented adaptive remeshing strategies (see Refs. [6–9]). The work presented here also provides a goal-oriented adaptive remeshing strategy based on a sensitivity analysis instead of the duality techniques.

The objective of the sensitivity analysis of a point wise error estimator is to know its derivative with respect to each of the nodal coordinates. This information provides a first order prediction of the variation of the point wise error estimator produced by a modification of the position of the nodal points. In this way, for a mesh with N nodal points the sensitivity analysis of a point wise error "e" provides the next vector of derivatives:

$$\frac{\mathrm{d}e}{\mathrm{d}\mathbf{x}} = \left[\frac{\mathrm{d}e}{\mathrm{d}x_1} \quad \frac{\mathrm{d}e}{\mathrm{d}y_1} \quad \frac{\mathrm{d}e}{\mathrm{d}x_2} \quad \frac{\mathrm{d}e}{\mathrm{d}y_2} \quad \cdots \quad \frac{\mathrm{d}e}{\mathrm{d}x_N} \quad \frac{\mathrm{d}e}{\mathrm{d}y_N}\right]^{\mathrm{T}}. \tag{1}$$

In this way, the first order prediction of the variation of the error that would be produced by an increment Δx_i of each nodal coordinate x_i would be:

$$\Delta e = \sum_{i=1}^{2N} \frac{\mathrm{d}e}{\mathrm{d}x_i}\Delta x_i = \frac{\mathrm{d}e}{\mathrm{d}\mathbf{x}}\Delta\mathbf{x}^{\mathrm{T}}. \tag{2}$$

In this work, the error e in expressions (1) and (2) corresponds with the estimated point wise error in the Von Mises stress $e^{\text{Von Mises}}$. This estimation is based on the difference between the finite element stresses and the smoothed stresses obtained from the SPR procedure. The corresponding expression for this point wise error estimator is the following (see Bugeda [3] for more details):

$$e^{\text{Von Mises}} = \sqrt{\frac{(\sigma_1^* - \sigma_2^*)^2 + (\sigma_2^* - \sigma_3^*)^2 + (\sigma_3^* - \sigma_1^*)^2}{6}} - \sqrt{\frac{(\hat{\sigma}_1 - \hat{\sigma}_2)^2 + (\hat{\sigma}_2 - \hat{\sigma}_3)^2 + (\hat{\sigma}_3 - \hat{\sigma}_1)^2}{6}}. \tag{3}$$

In expression (3), $\hat{\sigma}_i$ are the different principal components of the stress tensor obtained from the finite element analysis and σ_i^* are the corresponding values obtained from the SPR procedure.

This paper presents how the sensitivity analysis of the point wise error estimator can be computed and some of its possible applications including a new adaptive remeshing scheme.