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Sensitivity analysis for time dependent spatial price equilibrium problem

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Abstract

We present some sensitivity results for the spatial price equilibrium problem in the case of quantity formulation model and in presence of excess supply and excess demand. The equilibrium conditions that describe the above model are expressed in terms of a time dependent variational inequality. The variational inequality formulation plays a fundamental role in order to achieve the sensitivity results.

Keywords: Excess supply; Excess demand; Lagrangean function; Sensitivity analysis

1. Introduction

This paper is concerned with the sensitivity analysis of the equilibrium solution of the time dependent spatial price problem. The interest of this study resides in the fact that it is necessary to know how the solution changes under suitable variation of the data, for example, of the supply prices, the demand prices and the transportation costs. In order to perform this analysis it is useful to formulate this problem in terms of a time dependent variational inequality. In the static case the spatial price equilibrium problem has been formulated in terms of such a variational inequality by Nagurney and Zhao [7] and by Nagurney [6]. Subsequently Daniele [2] has considered the spatial price equilibrium problem in the case of the quantity formulation model under the assumption that the data evolve with the time. She proved that the time dependent equilibrium conditions can be directly incorporated into a time dependent variational inequality. In [5] the authors extended this result of [2] considering a model with excess supply and demand and with capacity constraints on prices and on transportation costs. In Section 2 we present the spatial price equilibrium problem in the case of the time dependent quantity formulation model and some existence theorems. In Section 3 we show how Lagrangean and duality theory can be applied to our problem and finally in Section 4 we give the sensitivity analysis results.

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2. A spatial price equilibrium problem

Let us consider n supply markets $P_i, i = 1, 2, \ldots, n$, and m demand markets $Q_j, j = 1, 2, \ldots, m$, involved in the production and in the consumption, respectively, of a commodity during a period of time $[0, T], T > 0$. Let $g_i(t), t \in [0, T], i = 1, 2, \ldots, n$, denote the supply of the commodity associated with supply market i at the time $t \in [0, T]$ and let $p_i(t), t \in [0, T], i = 1, 2, \ldots, n$, denote the supply price of the commodity associated with supply market i at the time $t \in [0, T]$. Fixed minimum and maximum supply prices $\underline{p}_i(t), \bar{p}_i(t) \geq 0$, respectively, for each supply market, are given. Let $f_j(t), t \in [0, T], j = 1, 2, \ldots, m$, denote the demand associated with the demand market j at the time $t \in [0, T]$ and let $q_j(t), t \in [0, T], j = 1, 2, \ldots, m$, denote the demand price associated with the demand market j at the time $t \in [0, T]$. Let $\underline{q}_j(t), \bar{q}_j(t) \geq 0$, for each demand market, be the fixed minimal and maximum demand prices, respectively. Since the markets are spatially separated, let $x_{ij}(t), t \in [0, T], i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$, denote the nonnegative commodity shipment transportated from supply market P_i to demand market Q_j at the same time $t \in [0, T]$. Let $c_{ij}(t), t \in [0, T], i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$, denote the nonnegative unit transportation cost associated with trading the commodity between (P_i, Q_j) at the same time $t \in [0, T]$. Let us suppose that we are in presence of excess supply and demand. Let $s_i(t), t \in [0, T], i = 1, 2, \ldots, n$, denote the excess supply for the supply market P_i at the time $t \in [0, T]$. Let $\tau_j(t), t \in [0, T], j = 1, 2, \ldots, m$, denote the excess demand for the demand market Q_j at the time $t \in [0, T]$. We assume that the following feasibility conditions hold for every $i = 1, 2, \ldots, n$, and $j = 1, 2, \ldots, m$, a.e. in $[0, T]$:

$$g_i(t) = \sum_{j=1}^{m} x_{ij}(t) + s_i(t), \tag{1}$$

$$f_j(t) = \sum_{i=1}^{n} x_{ij}(t) + \tau_j(t). \tag{2}$$

Grouping the introduced quantities in vectors, we have the total supply vector $g(t) \in L^2([0, T], \mathbb{R}^n)$ and the total demand vector $f(t) \in L^2([0, T], \mathbb{R}^m)$. Furthermore, in order to make precise the quantity formulation model, we assume that two mappings $p(g(t))$ and $q(f(t))$ are given:

$$p : L^2([0, T], \mathbb{R}^n) \to L^2([0, T], \mathbb{R}^n),$$
$$q : L^2([0, T], \mathbb{R}^m) \to L^2([0, T], \mathbb{R}^m).$$

The mapping p assigns for each supply $g(t)$ the supply price $p(g(t))$ and the mapping q assigns for each demand $f(t)$ the demand price $q(f(t))$. Analogously $x(t) \in L^2([0, T], \mathbb{R}^{nm})$ is the vector of commodity shipment and the mapping

$$c : L^2([0, T], \mathbb{R}^{nm}) \to L^2([0, T], \mathbb{R}^{nm})$$

assigns for each commodity shipment $x(t)$ the transportation cost $c(x(t))$. Moreover, let $s(t) \in L^2([0, T], \mathbb{R}^n)$ and $\tau(t) \in L^2([0, T], \mathbb{R}^m)$ be the vectors of excess supply and demand. Denoting by $w(t) = (g(t), f(t), x(t), s(t), \tau(t))$, we set

$$\tilde{L} = \{w(t) = (g(t), f(t), x(t), s(t), \tau(t)) : w(t) \in L^2([0, T], \mathbb{R}^n) \times L^2([0, T], \mathbb{R}^m) \times L^2([0, T], \mathbb{R}^{nm}) \times L^2([0, T], \mathbb{R}^n) \times L^2([0, T], \mathbb{R}^m)\},$$

and

$$\|w(t)\|_{\tilde{L}} = (\|g(t)\|^2_{L^2([0,T],\mathbb{R}^n)} + \|f(t)\|^2_{L^2([0,T],\mathbb{R}^m)} + \|x(t)\|^2_{L^2([0,T],\mathbb{R}^{nm})} + \|s(t)\|^2_{L^2([0,T],\mathbb{R}^n)} + \|\tau(t)\|^2_{L^2([0,T],\mathbb{R}^m)})^{1/2}.$$

Furthermore, we assume that the feasible vector $w(t) = (g(t), f(t), x(t), s(t), \tau(t))$ satisfies the condition

$$w(t) \geq 0 \quad \text{a.e. in } [0, T] \tag{3}$$