

# Sensitivity analysis and shape optimization for transient heat conduction with radiation

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## Abstract

Transient heat conduction problem is stated by the differential heat conduction equation, thermal boundary conditions on the external and internal boundary portions and the initial condition within the domain. Next an arbitrary behavioral functional is defined and its first-order sensitivities are determined using the material derivative concept as well as both the direct and adjoint approaches. The most used shape domain modifications are discussed in order to investigate the effect of design parameters on the integral radiation condition. The shape optimization problem is next formulated applying the obtained sensitivities. The illustration is the simple example of the shape optimization.

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## 1. Introduction

Radiative heat transfer is the important fundamental phenomena existing in practical engineering. The examples are the solar radiation in buildings, foundry engineering and solidification processes, die forging, chemical engineering, composite structures applied in industry. The physical analysis demonstrates that the radiative heat transfer problems are encountered as well in textiles (i.e. industrial textiles, textiles designed for use under hermetic protective barrier, multilayer clothing materials, etc.) as in textile structures (i.e. needle heating in heavy industrial sewing). Each of the above-mentioned radiative problems is the particular case characterized by a set of governing equations. Dems and Korycki [4] discuss some of these problems and give a short review of literature. The radiation within the hole is described here by the non-local integral condition according to Bialecki et al. [1,2]. The result is an integral equation describing the radiation intensification (caused by the reflected radiation) and absorbing of the

radiation within the isothermal and participating medium. These problems can be solved using different methods (cf. [15]). Roche and Sokolowski [14] gave also more information concerning numerical methods applying in optimization practice.

The presented paper is an extension of the steady problems stated and discussed by Dems and Korycki [4]. Other best general references here can be Dems and Mróz [3], Dems and Rousselet [6,7], and Korycki [10–13]. The first-order sensitivities of an arbitrary behavioral functional will be formulated as a function of the transformation velocity field and solutions of primary, direct and adjoint heat transfer problems, cf. [9].

The aim of this paper is to introduce the first-order sensitivities of an arbitrary behavioral functional to the shape design problems associated with the radiative heat transfer. A much more general modeling of transient conduction problems is considered here in view of radiative heat transfer on both the external and internal boundaries described by different conditions. These problems were not yet considered in the analyzed literature for the integral formulation of the radiation condition in transient problems.

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## Nomenclature

$a$	absorption coefficient of the radiation within the medium	$T_\infty$	surrounding temperature
$\mathbf{A}$	material conductivity matrix	$T_\infty^a$	adjoint surrounding temperature
$\mathbf{b}$	design parameters vector	$T_m$	temperature measured during the optimization on the boundary portion $\Gamma_m$
$c$	material heat capacity	$u$	unit cost of the material
$C$	structural cost, constraint in the shape optimization problem	$\mathbf{v}^p(\mathbf{x}, \mathbf{b}, t)$	transformation velocity field associated with the design parameter $b_p$
$C_0$	structural cost on the assumed level	$\mathbf{v}_n^p = \mathbf{n} \cdot \mathbf{v}^p$	transformation velocity normal to the boundary
$e_b(T) = \sigma T^4$	blackbody emissive power described by the Stefan–Boltzmann law	$\mathbf{x}$	vector of the coordinates
$f$	heat generation source of the primary structure	$\Gamma$	external boundary surrounding the domain $\Omega$
$f^a$	heat generation source of the adjoint structure	$\Gamma_T$	external boundary portion of the prescribed temperature
$F$	optional objective functional	$\Gamma_q$	external boundary portion of the prescribed heat flux density
$g_p = Dg/Db_p$	global (material) derivative of the function $g$ with respect to design parameter $b_p$	$\Gamma_c$	external boundary portion of the prescribed convective heat flux density
$g^p = \partial g / \partial b_p$	local (domain) derivative of the function $g$ with respect to design parameter $b_p$ calculated for the fixed domain $\Omega$	$\Gamma_d$	external boundary portion with the radiation
$h$	surface film conductance	$\Gamma_r$	internal boundary with the radiation
$H$	main curvature of the structural boundary $\Gamma$	$\varepsilon$	surface emissivity
$K(\mathbf{r}, \mathbf{p})$	kernel function of the radiation	$\sigma$	the Stefan–Boltzmann constant
$\mathbf{n}$	unit vector directed outwards on the boundary $\Gamma$	$\Sigma$	discontinuity line between two adjacent parts of the piecewise smooth boundary
$N$	number of objective functionals	$\tau$	time of the adjoint problem
$\mathbf{p}$	vector-coordinate of the observation point	$\Phi_p, \Phi_r$	angles between the line of sight and the normal to the surface directed outwards on $\Gamma$ at the observation and the current points, respectively
$P$	number of design parameters	$\varphi(\mathbf{x}, \mathbf{b}, t)$	given function of the space $\mathbf{x}$ , the design parameters $\mathbf{b}$ , and the time $t$
$q_n = \mathbf{n} \cdot \mathbf{q}$	heat flux density normal to the boundary	$\chi$	Lagrange multiplier which is an optional real number
$\tilde{q}_n^r = \mathbf{n} \cdot \tilde{\mathbf{q}}^r$	radiative heat flux density normal to the internal boundary $\Gamma_r$	$\omega$	rotation vector characterizing the rotation of the domain
$\mathbf{q}$	heat flux density vector	$\zeta^2$	additional variable (slack variable) in Lagrange functional
$\mathbf{q}^a$	heat flux density vector of the adjoint structure	$\Omega$	thermal anisotropic domain of the structure
$\mathbf{q}^*$	initial heat flux density vector	$\nabla$	gradient operator
$\mathbf{q}^{*a}$	initial heat flux density vector of the adjoint structure	$(\cdot)_{,\Delta}$	local derivative of the adequate function $(\cdot)$ with respect to $\Delta$
$\mathbf{r}$	vector-coordinate of the current point	$(\cdot)_p$	global derivative of the adequate function $(\cdot)$ with respect to design parameter $b_p$
$t$	time of the primary and additional problem		
$T$	state variable of the primary problem, the temperature field		
$T^a$	state variable of the adjoint problem, the temperature field within adjoint structure		
$T^p$	state variable of the additional problem associated with design parameter $b_p$		

## 2. Primary heat conduction problem

Let us introduce the transient heat conduction problem within a thermal anisotropic domain  $\Omega$  bounded by the boundary  $\Gamma$  (Fig. 1). The state variable is now the temperature  $T$ . The radiation on the part  $\Gamma_d$  of the external boundary is stated using the Stefan–Boltzmann law. The boundary portion  $\Gamma_r$  is an internal boundary and the radiation can be described using the most general form of radi-

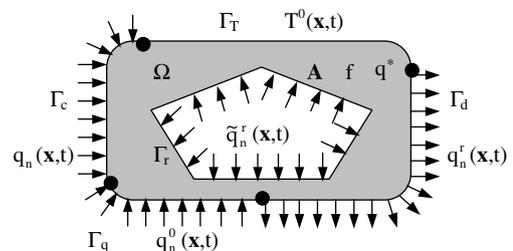


Fig. 1. Primary heat conduction problem.

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