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Sensitivity Analysis in Multiobjective Differential Programming

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Abstract—In this paper we deal with sensitivity analysis in multiobjective differential programs with equality constraints. We analyze the quantitative behavior of the optimal solutions according to changes of right-hand side values included in the original optimization problem. One of the difficulties lies in the fact that the efficient solution in multiobjective optimization in general becomes a set. If the preference of the decision maker is represented by a scalar utility which transforms optimal solutions in optima, we may apply existing methods of sensitivity analysis. However, when dealing with a subset of optimal points, the existence of a Fréchet differentiable selection of such a set-valued map is usually assumed. The aim of the paper is to investigate the derivative of certain set-valued maps of efficient points. We show that the sensitivity depends on a set-valued map associated to the T-Lagrange multipliers and a projection of its sensitivity.

1. INTRODUCTION

The main objective of the present article is to analyze sensitivity in vectorial optimization programs, that is, to study the variations of a set of efficient points when a certain parameter of the program varies.

There is a very important fact that marks an essential difference between scalar and vectorial programming. Whereas in the case of scalar programming the optimal point reached is a minimum point, and therefore unique, in the case of vectorial programming the optimal ones are minimal points. This implies that, in general, it turns out to be more complicated to analyze sensitivity in vectorial optimization programs than in scalar optimization programs, because whilst in the case of scalar programming the analysis of sensitivity consists of the study of a function (the function that assigns to each value of some parameter the optimal point reached by its associated program), in the case of vectorial programming, the analysis of sensitivity may necessitate the study of a set-valued map (the set-valued map that assigns to each value of some parameter the set of optimal points reached by its associated program).

One of the techniques used in sensitivity analysis is to reduce the problem by choosing a particular point in the efficient line. This is the case if we are interested in the best alternative which minimizes a specific scalar utility function as in [1] where the authors reduce to an optimization problem with scalar objective by minimizing the distance between some fixed desirable point and the efficient set or in [2] where the scalarization is done by the weighted sum approach, etc.

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When dealing with a subset or the whole set of efficient points, there are several procedures. One is to assume the existence of an adequate selection of particular efficient points as in [3] where the authors study sensitivity taking a selection of the balanced points introduced by Galperin and further developed in [4]. In [5–8] and [9] the authors consider the so-called T-optimal solutions and also assume the existence of a Fréchet differentiable selection. However, there are several approaches which deal with sets of efficient points and focus on the behaviour of some set-valued perturbation maps (e.g., [10–12], the two survey papers [13] and [14], and the references therein).

Continuing the line of inquiry of [7], sensitivity analysis will be performed for a differential vector program with equality constraints with respect to the right-hand side. Here we study the derivative of the set-valued perturbation map which deals with the T-optimal solutions.

The paper is organized as follows. Section 2 introduces notation, basic concepts, and some results that will be used throughout the paper. Section 3 is devoted to identify some conditions of regularity that allow the extension of some useful properties of Fréchet differentiable functions to derivable set-valued maps. Theorems 6 and 8 constitute the main results obtained in this section. Sensitivity analysis is the aim of Section 3. Namely, Theorem 12 states that the sensitivity of the problem depends not only on a suitable Lagrange multiplier but also on the derivative of a set-valued function of Lagrange multipliers.

2. PRELIMINARIES AND NOTATIONS

We recall some basic definitions and facts dealing with set-valued maps that we will use throughout the paper. For further information about set-valued analysis see, for instance, the book of [15].

Let S_1 and S_2 be two normed spaces, $A \subset S_1$ a nonempty set, and $\bar{A}$ its closure in the norm topology.

DEFINITION 1. *Let $x \in \bar{A}$. The contingent cone $T_A(x)$ is defined by*

$$T_A(x) = \left\{ v \in S_1 \;\middle|\; \liminf_{h \to 0+} \frac{d(A, x + hv)}{h} = 0 \right\}.$$

The adjacent cone $T_A^\flat(x)$ is defined by

$$T_A^\flat(x) = \left\{ v \in S_1 \;\middle|\; \liminf_{h \to 0+} \frac{d\,(A, x + hv)}{h} = 0 \right\}.$$

The set A is said to be derivable at x whenever the equality $T_A(x) = T_A^\flat(x)$ holds.

It is very convenient to recall the following characterizations of the above cones.

PROPOSITION 2. *Let $x \in \bar{A}$ and $v \in S_1$.*

(i) *$v \in T_A(x)$ if and only if there exist sequences $\{h_n\}_{n=1}^\infty \subset \mathbb{R}_+ \setminus \{0\}$ converging to 0 and $\{v_n\}_{n=1}^\infty \subset S_1$ converging to v, such that*

$$x + h_n v_n \in A$$

for all $n \in \mathbb{N}$.

(ii) *$v \in T_A^\flat(x)$ if and only if for any sequence $\{h_n\}_{n=1}^\infty \subset \mathbb{R}_+ \setminus \{0\}$ converging to 0, there exists a sequence $\{v_n\}_{n=1}^\infty \subset S_1$ converging to v such that*

$$x + h_n v_n \in A$$

for all $n \in \mathbb{N}$.