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Imperfection sensitivity analysis of hill-top branching with many symmetric bifurcation points

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Abstract

Imperfection sensitivity properties are derived for finite dimensional elastic conservative systems exhibiting hill-top branching at which arbitrary many bifurcation points coincide with a limit point. The critical load at a hill-top branching point is demonstrated to be insensitive to initial imperfections when all the bifurcation points are individually symmetric. Therefore, it is not dangerous to design a frame or truss so that many members buckle simultaneously at the limit point, although the notion of the danger of optimization by compound bifurcation is widespread.

Keywords: Coincident buckling; Hill-top branching; Imperfection sensitivity; Member buckling; Symmetric bifurcation

1. Introduction

The simultaneous buckling was studied in association with optimization. The principle of simultaneous mode design states, "A given form will be optimum if all failure modes which can possibly intersect occur simultaneously (Spunt, 1971)". The danger of naive optimization without due regard to imperfection sensitivity and the erosion of optimization by compound branching were suggested (Thompson and Supple, 1973). Various kinds of structures were found highly imperfection-sensitive when two or more bifurcation points are nearly or strictly coincident, and are subjected to interaction of buckling modes, such as local and global modes (Hutchinson and Amazigo, 1967; Koiter and Kuiken, 1971; Thompson and Lewis,

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1972; Tvergaard, 1973). Thompson and Hunt (1974) suggested extreme enhancement of imperfection sensitivity due to modal interaction as a result of optimization; imperfection sensitivity of coincident critical points was studied thereafter (Thompson and Hunt, 1984; Hunt, 1986).

Yet such severe enhancement of imperfection sensitivity is absent for another kind of coincident critical points. A nearly coincident pair of a bifurcation point and a limit point of loading parameter was found in (a) numerical simulation of a long tensile steel specimen undergoing plastic instability (Needleman, 1972), and (b) mechanical instability of stressed atomic crystal lattices (Thompson and Schorrock, 1975). Such a pair of points was approximated by a hill-top branching (bifurcation) point, at which the pair of points coincide strictly. This hill-top point was shown to enjoy locally piecewise linear imperfection sensitivity (Thompson and Schorrock, 1975; Thompson, 1982; Ikeda et al., 2002; Okazawa et al., 2002), which is less severe than the two-thirds power-law for a simple pitchfork bifurcation point. A piecewise linear relationship was also observed for other hill-top branching points that occur as the coincidence of

(i) an asymmetric bifurcation point and a limit point (Ohsaki, 2003), and
(ii) a limit point and a double bifurcation point studied by a group-theoretic approach (Ikeda et al., 2005).

Ohsaki (2000) optimized shallow trusses under constraints on nonlinear buckling and found that the optimum solution usually has a hill-top branching point, which is not sensitive to imperfections. Thus the optimization for nonlinear buckling does not always produce a dangerous structure.

It is noteworthy that, for a pin-jointed truss, member buckling can occur almost independently from global buckling (Peek and Triantafyllidis, 1992). Therefore, it is possible to create a hill-top branching point at which arbitrary many symmetric bifurcation points can exist at a limit point; i.e. many members buckle simultaneously with global buckling.

The basic framework to deal with coincident critical points can be found in the static perturbation method (Supple, 1967, 1968; Thompson and Hunt, 1973, 1984; Godoy, 1999); in this framework, compound bifurcations were studied in detail (Hunt, 1981). Critical points can be classified by investigating the linear, quadratic, cubic, quartic, etc., terms of the total potential energy (Thompson and Hunt, 1973). The interaction between bifurcation modes is classified into third-order and fourth-order interactions; the third-order interaction exists if one of the bifurcation modes is asymmetric. The maximum load of an imperfect symmetric system is reduced if the fourth-order cross-term is negative (Thompson and Hunt, 1984).

In this paper, imperfection sensitivity of a hill-top branching point with many symmetric bifurcation points is investigated. This point is actually created for a pin-jointed truss with simultaneously buckling members. The bifurcation modes are individually symmetric among themselves, but some modes have infinitesimally small third-order interaction. The symmetry conditions with respect to bifurcation modes and limit-point-type mode are relaxed by ignoring such interaction to account for practical situation of member buckling at the limit point.

2. Illustrative example of a hill-top branching

We start with a simple illustrative example: a two-bar truss as shown in Fig. 1, where $H = 100$ mm and $L = 1000$ mm. The members are connected to the nodes by pin joints. Each member is divided into four beam elements to implement member buckling. Green's strain is used for representing geometrically nonlinear strain–displacement relation. In the following, the units of length and force are mm and kN, respectively.

Maple 9, a symbolic computation software, is used for the differentiation of the total potential energy with respect to displacements, imperfection parameters, etc. The equilibrium paths are traced by