

# Mathematical model and sensitivity analysis for helical groove machining

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## Abstract

In any drill design, the helical groove shape plays a key role in ensuring an adequate flute space and an efficient chip removal capability. Moreover, the shape of the helical groove determines the principal drill angles. This study establishes a mathematical model of the helical groove and conducts a sensitivity analysis for helical groove machining performed on a 6-axis tool-grinding machine. Combining homogenous coordinate transformation and conjugate surface theory, a kinematic model is developed to facilitate the design of the helical groove shape (a direct problem). In determining the tool profile required to generate a desired helical groove (an inverse problem), this study exploits the condition that the common normal at the contact point between the tool and the helical groove surface must intersect the axis of the tool. The sensitivity of the helical groove profile with respect to the machining parameters is investigated. Finally, numerical examples are provided to demonstrate the validity of the developed models and algorithms. The numerical results reveal that the current design and sensitivity analysis methodology is comprehensive, simple and applicable to a wide range of helical groove machining applications.

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## 1. Introduction

Helical groove machining is the process by which a helically swept groove is generated on a cylindrical workpiece using a disk-type tool. The shape of the helical groove is of fundamental importance in any drill design since it determines the adequacy of the flute space and the efficiency of drill's chip removal capacity. The area of the helical flute must be sufficiently large to permit good chip removal, while the cross-section of the drill must be sufficiently rigid to withstand the cutting conditions. Moreover, the shape of the helical flute determines the rake angle, helix angle and inclination angle of the drill. The geometry of the machined helical groove depends both on the geometry of the tool and on the machining parameters employed. Therefore, it is necessary to develop a systematic modeling approach for the design and machining of helical grooves.

Constructing a mathematical model of the helical groove machining process involves two basic problems

[1]. The first problem involves determining the shape of the helical groove profile generated by a given tool (i.e. a direct problem), while the second deals with establishing the tool shape required to generate a particular helical groove transverse section profile (i.e. an inverse problem).

Due to the non-rectilinear motion of the tool along the helical cutting path, the geometry of the machined surface generated on the workpiece is not related simply to the cross-sectional profile of the cutting tool [2]. Recently, three different approaches have been developed for relating the tool profile to the helical profile. In the first method [2], the tool is regarded as being composed of infinitely thin disks of different diameters stacked side by side. The shape of the helical flute is developed as the envelope of the superimposed cutting paths of the individual disks. However, this approach fails to demonstrate the exact shape of the resulting helical groove. The second method [3] involves calculating the contact 3D space curve of the final grinding operation. In this method, the tool profile and the helical groove profile are both mathematically related to the contact curve. However, this method is unable to generate the tool surface uniquely because of the missing spans in the tool profile caused by numerical jumps at the corresponding sharp edges of the drill flute. In the third method, Kang et al.

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### Nomenclature

$(xyz)_0$	coordinate frame $(xyz)_0$ built in blank workpiece
$(xyz)_t$	coordinate frame $(xyz)_t$ built in grinding wheel
${}^0A_t$	configuration matrix of tool frame $(xyz)_t$ with respect to workpiece frame $(xyz)_0$
$r_{\text{groove}}$	helical groove surface
$R$	radius of blank workpiece

$\delta$	helix angle of helical groove
$r_t$	position vector of origin of tool frame with respect to workpiece frame $(xyz)_0$
$z_t$	unit vector of tool axis
${}^t r$	tool surface

[4] utilized the principles of differential geometry and kinematics to study the inverse and direct problems associated with helical groove machining. However, in their study, the engagement model, the geometry and kinematics relationship, and the cross-section of the helical groove were all based on the machine coordinates rather than the workpiece coordinates. Furthermore, the mathematical model was confined to a single tool orientation. Consequently, the model suffers a loss of generality, and is unsuitable for the case where the helical groove is to be machined on a 6-axis tool-grinding machine.

From the discussions above, it is clear that the existing models for helical groove machining tend to lack generality since they rely on restrictive machining methods, are designed for specific applications, and are difficult to implement in practice. Furthermore, a comprehensive mathematical model for helical groove machining on a 6-axis tool-grinding machine has yet to be reported in the literature. Therefore, this study systematically develops a generalized mathematical model for the design and machining of helical grooves.

The remainder of this study is organized as follows. Section 2 studies the revolution geometry of disk-type abrasive tools, while Section 3 uses conjugate surface theory to determine the helical groove profile. Section 4 considers the contact line between the tool and the groove surface and establishes the tool profile required to generate a particular helical groove. Section 5 examines the sensitivity of the helical groove with respect to the machining parameters. Section 6 provides numerical examples to demonstrate the validity of the developed models and algorithms. Finally, Section 7 presents some brief conclusions.

Throughout this study, the point vector  $a_x i + a_y j + a_z k$  is written in the form of the column matrix

$${}^j a = [a_x \quad a_y \quad a_z \quad 1]^T$$

It is noted that the pre-superscript  $j$  of the leading symbol, i.e.  ${}^j a$ , indicates that the vector is defined with respect to the coordinate frame  $(xyz)_j$ . Given a point  ${}^j a$ , its transformation,  ${}^k a$ , is represented by the matrix product  ${}^k a = {}^k A_j {}^j a$ , where  ${}^k A_j$  is a  $4 \times 4$  matrix defining the position and orientation (referred to hereafter as ‘configuration’) of the frame  $(xyz)_j$

with respect to another frame  $(xyz)_k$ . These notation rules are also applied to the unit directional vector, i.e.  ${}^j n = [n_x \quad n_y \quad n_z \quad 0]^T$ . For simplicity, if a vector is referred to the blank frame  $(xyz)_0$ , its pre-superscript, ‘0’ is omitted.

## 2. Tool model

In this study, the term ‘tool’ refers to high-speed rotational axis-type cutting tools or disc-type abrasive wheels. An important feature of such tools is that their working surfaces are surfaces of revolution. Consequently, these working surfaces can be studied in terms of their revolution geometry, and their unit normal and tangent vectors established. The tool surface,  ${}^t r$ , can be obtained by rotating the generating curve  ${}^t q = [x(h) \quad 0 \quad z(h) \quad 1]^T$  ( $x(h) > 0$  and parameter  $h$  varies over a specified range) in the  $x$ - $z$  plane about the symmetrical rotating  $z_t$  axis (see Fig. 1), i.e.

$${}^t r = \text{Rot}(z, v) {}^t q = [x(h)Cv \quad x(h)Sv \quad z(h) \quad 1]^T \quad (1)$$

where  $C$  and  $S$  denote cosine and sine, respectively, and  $\text{Rot}(z, v)$  is the rotation matrix about the  $z$ -axis. Eq. (1) is a generalized expression valid for parameterizing the working

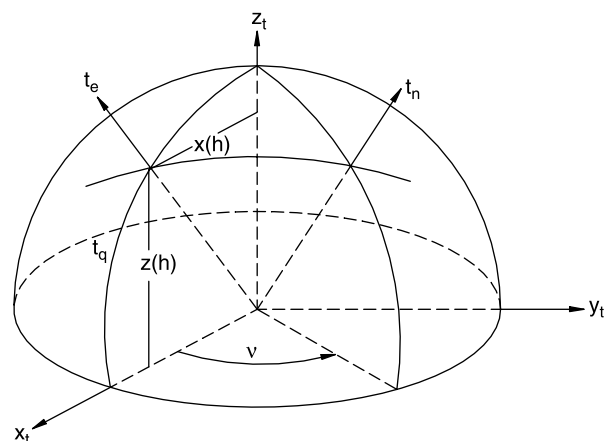


Fig. 1. Generating curve and its unit outward normal.

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