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Optimization and sensitivity analysis of space frames allowing for large deflection

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Abstract

A procedure for sensitivity analysis used with nonlinear incremental-iterative structural analysis of frames is proposed. The sensitivity of displacement and stress are considered. The accuracy and efficiency of this method are confirmed by several examples. The method can be used for the second-order analysis and optimization design of framed structures. Practical constraints and considerations for the design of steel frames are included in the present studies such that the reported findings can be used directly for practical optimal design, which is believed to have not yet been studied or reported in literatures.

Keywords: Sensitivity analysis; Nonlinear analysis; Space frames; Optimization

1. Introduction

Sensitivity analysis (SA) is a useful technique for the economical design of steel structures where the serviceability deflection limit state is a consideration. Its role is to evaluate the changes in structural response due to a variation in design parameters such as displacements, stresses and frequencies. Explicitly, the response derivatives are determined with respect to the design variables of sectional parameters. The sensitivity of the structural response with respect to these sectional design variables provides the designer with valuable insight into the structural response to these design variables.

Sensitivity analysis under linear analysis has been investigated extensively by many researchers. Chan [1] expressed the displacements of nodes explicitly in design variables using the principle of virtual work, and proposed a practical method for the optimization design of tall buildings. Adelman and Haftka [2] reviewed the general method for calculating the sensitivity of the static response, eigenvalues and eigenvectors, as well as the transient response. Their paper focused mainly on derivatives of the structural responses with respect to sectional variables such as cross-sectional area, second moment of area, and plate thicknesses.

For slender steel space frames, the linear relationships between the member forces and displacements become invalid because of the geometrical $P-\delta$ and $P-\Delta$ and material yielding nonlinear effects. As a result, traditional sensitivity analysis methods for linear structural behavior are not applicable. The simplest method for obtaining the derivatives of the structural response with respect to a design variable is the finite-difference method in both the linear and nonlinear cases. However, the computational cost is very high and it may be hard to find the appropriate step size in specific cases.

For nonlinear structural analysis, there are mainly two different types of solution method, namely the secant iterative method and the incremental-iterative method. The secant iterative method has the advantage of simplicity, in using only the secant stiffness relationships. The incremental-iterative method uses the tangent stiffness to estimate the displacement increments and secant stiffness to check the convergence. It has the general capability of traversing the limit point. A more specific comparison of the two methods has been made by Chan and Chui [3].

Ryu and Haririan [4] compared the difference between the sensitivity analysis procedures for linear and nonlinear

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responses. They further pointed out that the incremental-iterative approach is more appropriate for the design sensitivity analysis of nonlinear structures, as the tangent stiffness matrix at the final load level can be used directly. This advantage is also reported in this paper. A general procedure for design sensitivity analysis with incremental iterative nonlinear analysis of the structural systems is proposed by Tsay and Arora [5] in continuum formulations, and both the geometric and material nonlinearities are included in the derivations. But their approach appears to be less suitable for the optimization of nonlinear slender frames, which is the aim of this paper. These papers laid the foundation for the design optimization of complex nonlinear structures.

Xu [6] adopted a direct differentiation method (DDM) in the optimization of geometrical nonlinear and semi-rigid frames using the secant iteration solution method. Saka and Ulker [7] and Saka and Kameshki [8] used a pseudo-load technique to express the displacement by the element stiffness and assuming the reciprocal relationship between the displacement and the design variables from which they obtained the sensitivity of the displacement after differentiation. Pezeshk [9] used a similar technique, with the potential energy differentiated with respect to the design variables to obtain the sensitivity of the stiffness matrix. The error due to the lack of knowledge of the geometrical stiffness matrix will increase as the nonlinearity of the structure increases. Zhang [10] derived a sensitivity analysis method using the commercial software ABAQUS (5.5), but his method is only suitable for the bar and membrane elements, in which the element stiffness can be expressed in a separable form with the design variables, which is not applicable for beam–column or shell elements. From the review of optimization technique used in conjunction with nonlinear structural analysis, a sensitivity analysis method for the nonlinear framed structure suitable for use with the popular and robust incremental-iterative solution technique is not yet available in the literature.

In this paper, a procedure is proposed for sensitivity analysis designed for use with an incremental-iterative analysis method of nonlinear frames. As the design is for a serviceability limit state design with a moderate load factor, only geometrical nonlinearity is considered. Several examples reported here confirmed that this proposed procedure is accurate and efficient for the incremental-iterative type of structural analysis. The sensitivity computational cost is also noted to be nominal compared with the whole solution process. The usage of the proposed sensitivity analysis in practical structural optimization with the Optimality Criteria method is illustrated by an example of the optimization of a 15-story braced steel frame.

2. A review of the Newton–Raphson iterative procedure for nonlinear analysis

The well-known Newton–Raphson iterative method is the simplest technique for the effective solution of a nonlinear problem. Although this method cannot traverse the critical point due to the ill-conditioning of the Jacobian matrix, it is more appropriate for practical analysis and design before critical

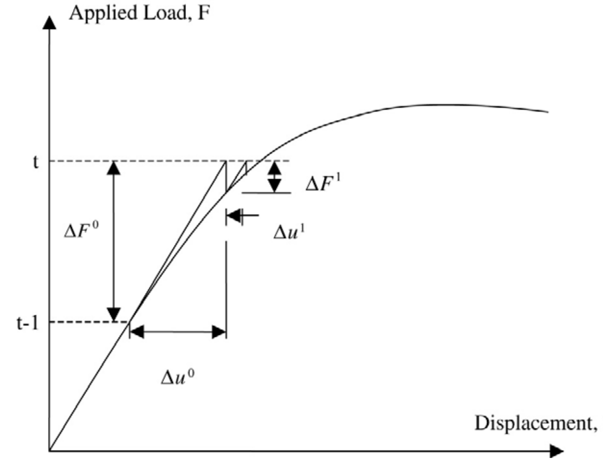

Fig. 1. Newton–Raphson iteration method.

load. Assuming a to be a k-dimensional design variable vector and U to be the vector of nodal displacements, the equilibrium equation can be written as,

$$R(a, U) - F(a) = 0 \tag{1}$$

in which $F(a)$ is the externally applied equivalent nodal load and $R(a, U)$ is the internal resistance nodal force. a is the design variable vector given by $a = (a_j, j = 1, \ldots, n)^{\mathrm{T}}$. The Newton–Raphson incremental-iteration equation can be written as,

$$\frac{\partial R}{\partial U}(a, {}^tU^{(i-1)})\Delta U^{(i)} = {}^tF(a) - {}^tR(a, {}^tU^{(i-1)}) \tag{2}$$

in which the left superscript t corresponds to the load level and the right superscript i represents the iteration number. Note that the global tangent stiffness matrix, ${}^tK_T^{(i-1)}$, is given by,

$$\frac{\partial R}{\partial U}(a, {}^tU^{(i-1)}) = {}^tK_T^{(i-1)}. \tag{3}$$

The displacement increment $\Delta U^{(i)}$ can be obtained by solving Eq. (2). Adding the displacement increment to the displacement of the last iteration, we obtain,

$${}^tU^{(i)} = {}^tU^{(i-1)} + \Delta U^{(i)}. \tag{4}$$

The process from Eqs. (2) (4) continues until convergence, as illustrated in Fig. 1.

From Eq. (2), we can see that the tangent stiffness matrix should first be formed in the complete iterative process. The secant relationship should also be used to calculate the unbalanced forces during iteration, which is the right-hand part of Eq. (2).

3. Secant and tangent stiffness relationships

The simplest element for obtaining the nonlinear secant and tangent stiffness relationships is to extend the cubic Hermite element by including the geometric stiffness within the linear stiffness matrix to form the tangent stiffness matrix. This approach has been used by many researchers (see, [11–13]). This method is quite successful, except for the necessity of