Available online at www.sciencedirect.com
SCIENCE DIRECT®
Structural Safety 28 (2006) 321–340
STRUCTURAL SAFETY
www.elsevier.com/locate/strusafe

Optimal cost design with sensitivity analysis using decomposition techniques. Application to composite breakwaters

Roberto Mínguez [a,b,*], Enrique Castillo [c], Carmen Castillo [d], Miguel A. Losada [d]

[a] *Department of Applied Mathematics, University of Castilla-La Mancha, 13071 Ciudad Real, Spain*
[b] *Department of Statistics, Cornell University, Ithaca, NY 14853-3901, USA*
[c] *Department of Applied Mathematics and Computational Sciences, University of Cantabria, Avda. Castros s/n., 39005 Santander, Spain*
[d] *Grupo de Puertos y Costas, CEAMA, University of Granada, Avda. del Mediterráneo s/n, 18071 Granada, Spain*

Received 25 May 2004; received in revised form 9 May 2005; accepted 8 August 2005
Available online 13 October 2005

Abstract

Minimizing the expected total cost of a structure, including maintenance and construction is a difficult problem because of the presence in the objective function of the yearly failure rates, which have to be calculated by an optimization problem per each failure mode. In this paper, a new method for the design of structures that minimizes the total expected costs of the structure during its lifetime based on Benders' decomposition is presented. In addition, some tools for sensitivity analysis are introduced, which make it possible to determine how the cost and yearly failure rates of the optimal solution are affected by small changes in the input data values. The proposed method is illustrated by its application to the design of a composite breakwater under breaking and non-breaking wave conditions.

Keywords: Decomposition techniques; Benders decomposition; Cost optimization; Reliability-based design; Failure probability; Modes of failure

1. Introduction

Engineering design of structural elements is a complicated and highly iterative process that usually requires an extensive experience. Iterations consist of a trial-and-error selection of the design variables or parameters, together with a check of the safety and functionality constraints, until reasonable structures, in terms of cost and safety, are obtained. Since maintenance and repair take place during the service lifetime of the structure, the associated costs must be added to construction costs. The objective of the design is to verify that the

* Corresponding author. Tel.: +34 926295300(6218); fax: +34 926295391.
E-mail addresses: Roberto.Minguez@uclm.es, rm333@cornell.edu (R. Mínguez), castie@unican.es (E. Castillo), mcastill@ugr.es (C. Castillo), mlosada@ugr.es (M.A. Losada).

0167-4730/$ - see front matter
doi:10.1016/j.strusafe.2005.08.005

structure satisfies the project requirements during its lifetime in terms of acceptable failure rates and cost (see Losada [1] and ROM [2]).

Since repair depends on the modes of failure and their frequencies, these must be defined. Each mode of failure m is defined by its corresponding limit state equation as, for example:

$$g_m(x_1, x_2, \ldots, x_n) = h_{sm}(x_1, x_2, \ldots, x_n) - h_{fm}(x_1, x_2, \ldots, x_n); \quad m \in M, \tag{1}$$

where $(x_1, x_2, \ldots, x_n)$ refer to the values of the variables involved, $g_m(x_1, x_2, \ldots, x_n)$ is the safety margin and $h_{sm}(x_1, x_2, \ldots, x_n)$ and $h_{fm}(x_1, x_2, \ldots, x_n)$ are two opposing magnitudes (such as stabilizing and mobilizing forces, strengths and stresses, etc.) that tend to avoid and produce the associated mode of failure, respectively, and M is the set of all failure modes.

The failure occurs when the critical variables satisfy $g_m \leqslant 0$. With the consideration of all extreme events (see [3,4]) that may occur in the reference period, the different failure rates for all failure modes can be estimated.

Over the last few years, design methods have been improved by applying optimization techniques. The main advantage is that these techniques lead to optimal design and automation. Designer's concerns are only the constraints to be imposed on the problem and the objective function.

Some authors consider the construction cost [5–9] or the total cost (construction, maintenance and repairs) as the design criteria [10–14]. The main problem of including repair and maintenance cost is that in such a case the cost function includes yearly failure rates, the calculation of which implies solving as many optimization problems as failure modes. Thus, use of optimization programs is not straightforward.

In addition to requiring optimal solutions to problems, some interest is shown by people in knowing how sensitive are the solutions to data values. A sensitivity analysis provides excellent information on the extent to which a small change in the parameters or assumptions (data) modifies the resulting design (geometric dimensions, costs, reliabilities, etc.).

The aims of this paper are: (a) to present a decomposition design method that permits solving the total cost minimization problem and (b) to provide tools to perform a sensitivity analysis.

The paper is structured as follows. In Section 2, the proposed method for optimal design based on Benders' decomposition is presented. In Section 3, a technique for performing a sensitivity analysis is explained. Section 4 illustrates the proposed method by an example dealing with the design of a composite breakwater. Section 5 is devoted to the discussion of the statistical assumptions. Section 6 presents a numerical example. Finally, Section 7 gives some conclusions.

2. Proposed method for optimal design

In the design and reliability analysis of a structure, there are some random variables $(X_1, \ldots, X_n)$ involved. They include geometric variables, material properties, loads, etc. In this paper, without loss of generality, we make no distinction between random and deterministic variables. So, deterministic variables are only particular cases of them.

It is important to distinguish between design values of the random variables X_i, and actual values x_i $(i = 1, 2, \ldots, n)$. The design values are those values used by the engineer at the design stage for the geometric variables (dimensions), the material properties (strengths, stiffness, etc.), that do not necessarily correspond with those in the real work. Thus, in this paper, the design values are assumed to be the means or the characteristic values (extreme percentiles) of the corresponding random variables, and are denoted $\bar{x}_i$ (mean) and $\tilde{x}_i$ (characteristic), respectively. Some of these design values are chosen by the engineer or given by the design codes, and some are selected by the optimization procedure to be presented. In this paper, the set of variables $(X_1, \ldots, X_n)$ will be partitioned in four sets:

1. *Optimized design variables* $\boldsymbol{d}$. Their mean values are to be chosen by the optimization procedure. Normally, they describe the dimensions of the work being designed, such as width, thickness, height, cross-sections, etc., but can include material properties, etc.