Scientia Iranica B (2013) 20 (2), 301–312

Sharif University of Technology

Scientia Iranica

Transactions B: Mechanical Engineering

www.sciencedirect.com

Maximum power output of multistage irreversible heat engines under a generalized heat transfer law by using dynamic programming

L. Chen *, **S. Xia**, **F. Sun**

College of Power Engineering, Naval University of Engineering, Wuhan 430033, PR China

Received 10 August 2011; revised 25 October 2012; accepted 21 January 2013

KEYWORDS

Generalized heat transfer law;
Multistage heat engine system;
Maximum power;
Optimal control;
Finite time thermodynamics.

Abstract A multistage irreversible Carnot heat engine system operating between a finite thermal capacity high-temperature fluid reservoir and an infinite thermal capacity low-temperature environment with a generalized heat transfer law $[q \propto (\Delta(T^n))^m]$ is investigated in this paper. Optimal control theory is applied to derive the continuous Hamilton-Jacobi-Bellman (HJB) equations, which determine the optimal fluid temperature configurations for maximum power output under the conditions of fixed initial time and fixed initial temperature of the driving fluid. Based on the universal optimization results, the analytical solution for the case with Newtonian heat transfer law ($m = 1, n = 1$) is further obtained. Since there are no analytical solutions for other heat transfer laws, the continuous HJB equations are discretized and the dynamic programming (DP) algorithm is performed to obtain the complete numerical solutions of the optimization problem. Then the effects of the internal irreversibility and heat transfer laws on the optimization results are analyzed in detail. The results obtained can provide some theoretical guidelines for the optimal design and operation of practical energy conversion and transfer processes and systems.

1. Introduction

In analysis of finite time thermodynamics or entropy generation minimization [1–15], the basic thermodynamic model is the endoreversible Newtonian law system in which only the irreversibility of the linear finite rate heat transfer is considered. Novikov [16], Chambadal [17], and Curzon and Ahlborn [18] first derived the maximum power output and corresponding efficiency of an endoreversible Carnot heat engine cycle with the Newtonian heat transfer law $[q \propto \Delta(T)]$, i.e. $\eta_{CA} = 1 - \sqrt{T_L/T_H}$, where T_H and T_L are the temperatures of heat source and heat sink, respectively. Yan [19] derived the relation between the optimal efficiency and optimal power output for an endoreversible Carnot heat engine, i.e. the fundamental optimal relation of the Carnot heat engine with the Newtonian heat transfer law. Sun et al. [20–22] obtained the holographic power versus efficiency spectrum, and formed the finite time thermodynamic optimization criteria for the parameter selection of an endoreversible Carnot heat engine with Newtonian heat transfer law. In general, heat transfer is not necessarily Newtonian. Gutowicz-Krusion et al. [23] first derived the maximum power and the corresponding thermal efficiency bound of an endoreversible Carnot heat engine with the generalized convective heat transfer law $[q \propto (\Delta T)^m]$. Some authors have assessed the effects of the linear phenomenological heat transfer law $[q \propto \Delta(T^{-1})]$ and radiative heat transfer law $[q \propto \Delta(T^4)]$ on the performance of endoreversible Carnot heat engines [24–28]. Chen et al. [29–31], Angulo-Brown and Paez-Hernandez [32] and Huleihil and Andresen [33] derived the optimal relation between power output and efficiency with the generalized convective heat transfer law [29,30,32,33] and mixed heat resistances [31]. De Vos [34,35] first derived the maximum power output and the corresponding efficiency of an endoreversible Carnot heat engine with the generalized radiative heat transfer law $[q \propto (\Delta T^n)]$. Chen and Yan [36] and Gordon [37] further derived the optimal relation between the power output and efficiency of the endoreversible Carnot heat engine, based on this heat transfer law. One of aims of finite time thermodynamics is to pursue generalized rules and results. Li et al. [38] derived the

* Corresponding author. Tel.: +86 27 836 15046; fax: +86 27 836 38709.
E-mail addresses: lingenchen@hotmail.com, lgchenna@yahoo.com (L. Chen).

Peer review under responsibility of Sharif University of Technology.

Production and hosting by Elsevier

1026-3098 © 2013 Sharif University of Technology. Production and hosting by Elsevier B.V. All rights reserved.
doi:10.1016/j.scient.2013.02.002

optimal relation between the power output and efficiency of an endoreversible Carnot heat engine with a generalized heat transfer law [$q \propto (\Delta(T^n))^m$], in which the results in Refs. [16–37] were included. However, the work mentioned above was restricted to static optimization research into a class of single-stage steady systems, and the optimization methods used are also very simple. Since the mid 1990s, dynamic optimization researches on complex multistage unsteady thermodynamic systems using the Hamilton–Jacobi–Bellman (HJB) theory has always been a very important research field in finite time thermodynamics. Sieniutycz [5,7,11,39–45], Sieniutycz and Spakovsky [46], and Szwast and Sieniutycz [47] investigated the maximum power output of Newtonian law multistage continuous endoreversible Carnot heat engine systems, operating between a finite thermal capacity high-temperature fluid reservoir and an infinite thermal capacity low-temperature environment, by applying the HJB theory [5, 7,11,39–43,46], and the results were further extended to the multistage discrete endoreversible Carnot heat engine system [5,7,11,44,45,47]. Li et al. [48] further considered that both high- and low-temperature sides are finite thermal capacity fluid reservoir, and investigated the problems of maximizing the power output of multistage continuous endoreversible Carnot heat engine systems with the Newtonian heat transfer law. Xia et al. [49,50] further considered that the heat transfer processes obeyed $q \propto (\Delta T)^m$ [49] and $q \propto (\Delta(T^n))^m$ [50] heat transfer laws, respectively, and investigated the maximum power output of a multistage continuous endoreversible Carnot heat engine system with the finite thermal capacity high-temperature fluid reservoir by applying the HJB theory.

However, real heat engines are usually devices with internal and external irreversibilities. Besides the irreversibility of finite rate heat transfer, there are also other sources of irreversibilities, such as the bypass heat leakage, dissipation processes inside the working fluid, etc. Some authors have assessed the effects of finite rate heat transfer, together with major irreversibilities, on the performance of heat engines using the heat resistance and bypass heat leakage model [24,51–53], heat resistance and internal irreversibility model [54]. Chen [9], Chen and Sun [55] and Chen et al. [56] established a generalized irreversible Carnot heat engine model which took account of the effect of heat resistance, bypass heat leakage and other internal irreversibility, and derived its optimal relation between power output and efficiency with the Newtonian heat transfer law. Chen et al. [57] and Zhou et al. [58] further derived the fundamental optimal relation between the power output and efficiency of the generalized irreversible Carnot heat engine with generalized radiative [57] and generalized convective [58] heat transfer laws, respectively. Chen et al. [59] derived the optimal relation between the power output and efficiency of a generalized irreversible Carnot heat engine with heat transfer law $q \propto (\Delta(T^n))^m$, in which the results in Refs. [9, 16–38,51–58] were included. Sieniutycz and Szwast [60] and Sieniutycz [11,61] further investigated the maximum power output of a multistage irreversible Carnot heat engine system with a finite high-temperature heat reservoir and the Newtonian heat transfer law, and the irreversibility includes the heat resistance and internal dissipation. Li et al. [62] further investigated the problems of maximizing the power output of multistage continuous irreversible Carnot heat engine system with two finite thermal capacity heat reservoirs and Newtonian heat transfer law. Sieniutycz and Kuran [63,64], Kuran [65] and Sieniutycz [11,66–69] investigated the maximum power output of a finite high-temperature fluid reservoir multistage

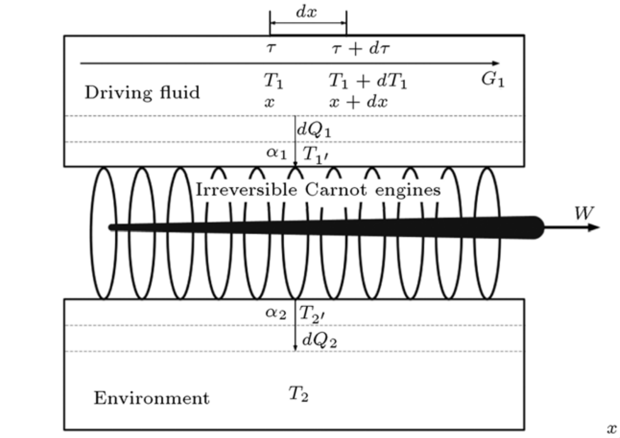

Figure 1: Model of multistage continuous irreversible Carnot heat engine system with finite high-temperature fluid reservoir.

continuous irreversible Carnot heat engine system with the radiative heat transfer law. Because there are no analytical solutions for cases with the pure radiative heat transfer law, the authors of Refs. [11,64–69] obtained the analytical solutions of the optimization problems by replacing the radiative heat transfer law by the so called pseudo-Newtonian heat transfer law [$q \propto \alpha(T^3)(\Delta T)$] approximately, which is the Newtonian heat transfer law with a heat transfer coefficient, $\alpha(T^3)$, as a function of the cube of the fluid reservoir temperature. Li et al. [70] further investigated the problems of maximizing the power output of a multistage continuous endoreversible Carnot heat engine system with two finite thermal capacity heat reservoirs and the pseudo-Newtonian heat transfer law. Sieniutycz [71] further investigated the maximum power output of the multistage continuous irreversible Carnot heat engine system with the non-linear heat transfer law [$q \propto \alpha(T^n)(\Delta T)$], i.e. the Newtonian heat transfer law with a heat transfer coefficient $\alpha(T^n)$ as a function of the n-power of the fluid reservoir temperature. Xia et al. [72] investigated the maximum power output of a multistage continuous irreversible Carnot heat engine system with generalized convective heat transfer law by applying the HJB theory.

Based on Refs. [5,7,11,39–50,60–72], this paper will further investigate the maximum power output of a multistage irreversible Carnot heat engine system, in which the heat transfer between the reservoir and the working fluid obeys the generalized heat transfer law [$q \propto (\Delta(T^n))^m$] [38,50,59,73–78], and the irreversibility includes the heat resistance and internal dissipation.

2. System modeling and characteristic description

The model of a multistage continuous irreversible Carnot heat engine system with a finite high-temperature fluid reservoir to be considered is shown in Figure 1. The first fluid (i.e. the driving fluid) flows along the x-axis, and the infinitesimal Carnot heat engines are located continuously between two separated boundary layers of the fluids. Each infinitesimal Carnot heat engine is the same. During the infinitesimal length, dx, the infinitesimal Carnot heat engine absorbs heat from the first fluid, and releases heat to the second fluid (i.e. environment). The cumulative power is delivered at the last stage. The thermal capacity of the high-temperature fluid is finite, and its temperature decreases along the flow