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Charge sensitivity analysis of multiple discharges in a three conductor system with small gaps ☆

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Abstract

The concept of charge sensitivity factors is introduced for analysis of electrostatic discharge (ESD) events. The methodology is used to study the conditions for multiple discharges for an in-line three cylinder geometry with small gaps. The method can be applied to any structure with multiple fixed gaps where charge results from motion with respect to a reference structure.

Keywords: ESD; Charge sensitivity; Three body problem

1. Introduction

Most electrostatic discharge (ESD) events are modelled as the approach of a charged floating body (the source) to another body (the sink) which can be either floating or grounded. In the human body model (HBM), the source is the charged human body; in the charged device model (CDM), the source is the charged integrated circuit. Triboelectrification and induction charging are the prime charge generation methods. During the final approach of the source to the sink, the charge on the source body is usually assumed to be constant; the change in body potentials and the probability of a discharge event are due to changes in the capacitances of the bodies in the given geometry.

Recently, this analysis was extended to include a three body problem with small gaps [1]; a system of bodies with fixed charge and changing separation was studied to determine conditions for primary and secondary discharges [2,3]. A review of the literature showed an increase in interest to study low-voltage discharges, including multiple discharges, in small gap geometries [4–16].

In this work, the analysis is extended to include a fixed geometry of conductors with body charge being the variable. In application, this may occur in a geometry in which gaps between a number of stationary conductors is fixed; charging of the system conductors could occur due to linear or rotational motion with respect to another reference body; possible charging mechanisms include triboelectrification and induction charging.

2. Three body problem

The general three body problem is shown in Fig. 1. In the most general case, all three bodies are floating and charged; Maxwell's method to formulate multi-body capacitances for a system of conductors can be expressed as

$$Q_1 = c_{11}V_1 + c_{12}V_2 + c_{13}V_3, \quad (1)$$

$$Q_2 = c_{21}V_1 + c_{22}V_2 + c_{23}V_3, \quad (2)$$

$$Q_3 = c_{31}V_1 + c_{32}V_2 + c_{33}V_3, \quad (3)$$

where Q_i is the charge on body i; V_i is the potential of body i; c_{ii} is the self-capacitance coefficient of body i; c_{ij} is the mutual capacitance coefficient between body i and body j.

For experimentally determined values of capacitance coefficients and assumed values of body charges, body potentials can be solved and potential differences V_{12} and V_{23} calculated; for the experimental geometry studied,

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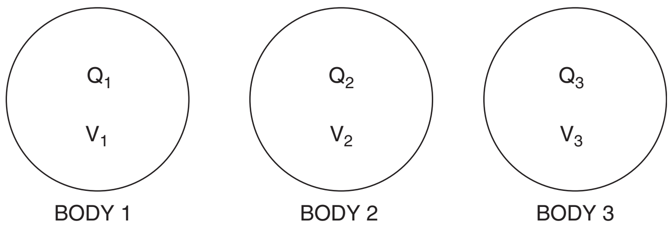

Fig. 1. Generalized three body problem.

these are the only potential differences of interest since they represent the two cases of neighbouring conductors. Differentiation of the expressions for V_{12} and V_{23} with respect to the system charges will yield a series of charge sensitivity factors which are defined as follows:

$$S_{12\text{-}1} = \frac{\partial V_{12}}{\partial Q_1}, \tag{4}$$

$$S_{12\text{-}2} = \frac{\partial V_{12}}{\partial Q_2}, \tag{5}$$

$$S_{12\text{-}3} = \frac{\partial V_{12}}{\partial Q_3}, \tag{6}$$

$$S_{23\text{-}1} = \frac{\partial V_{23}}{\partial Q_1}, \tag{7}$$

$$S_{23\text{-}2} = \frac{\partial V_{23}}{\partial Q_2}, \tag{8}$$

$$S_{23\text{-}3} = \frac{\partial V_{23}}{\partial Q_3}. \tag{9}$$

Assume a first discharge event occurs between body 1 and body 2. During the discharge, the original charges, Q_1 on body 1 and Q_2 on body 2, are distributed between the two bodies which are now at a common potential V; the new body 1 and body 2 charges are q_1 and q_2, respectively. The system equations become

$$q_1 = c_{11}V + c_{12}V + c_{13}V_3, \tag{10}$$

$$q_2 = c_{21}V + c_{22}V + c_{23}V_3, \tag{11}$$

$$Q_3 = c_{31}V + c_{32}V + c_{33}V_3. \tag{12}$$

By charge conservation, $Q_1 + Q_2 = q_1 + q_2$. Eqs. (10)–(12) can be solved to yield V and V_3 and the new charges q_1 on body 1 and q_2 on body 2; after the discharge event, Eqs. (1)–(3) can be solved to yield the new body potentials.

Alternately, assume a first discharge event between body 2 and body 3. During the discharge, the original charges, Q_2 on body 2 and Q_3 on body 3, are distributed between the two bodies which are at a common potential V'; the new body 2 and body 3 charges are q'_2 and q'_3, respectively. The system equations become

$$Q_1 = c_{11}V_1 + c_{12}V' + c_{13}V', \tag{13}$$

$$q'_2 = c_{21}V_1 + c_{22}V' + c_{23}V', \tag{14}$$

$$q'_3 = c_{31}V_1 + c_{32}V' + c_{33}V'. \tag{15}$$

By charge conservation, $Q_2 + Q_3 = q'_2 + q'_3$. Eqs. (13)–(15) can be solved to yield V' and V_1 and the new charges q'_2 on body 2 and q'_3 on body 3; after the discharge event, Eqs. (1)–(3) can be solved to yield the new body potentials.

For both of these first discharge cases, the solution of the system equations yields the new distribution of charge between the system conductors, assuming conservation of charge. After the first discharge, the system equations can be solved again to yield changes in body potentials due to the new distribution of charge on the conductors.

3. Experimental method

In this work, an assessment of the potentials of a three cylinder system located away from ground planes was performed. The gaps between the conductors were fixed and body charge considered as a variable. In application, this situation could occur in a manufacturing environment in which a platform of closely spaced conductors is moving with respect to another reference body and charging takes place due to triboelectrification. Three aluminum cylinders of equal height (30.5 cm) and different diameters were used; the following nomenclature was used for the various sizes: S (small)—diameter 6.0 cm; M (medium)—diameter 11.4 cm; L (large)—diameter 20.3 cm. The experimental arrangement is shown in Fig. 2. The three cylinders, referred to as body 1, body 2 and body 3, were positioned as shown with their centre lines in parallel and capacitance coefficients were determined for a constant separation between body 1 and body 2 ($d_{12} = 0.25$ mm) and a constant separation between body 2 and body 3 ($d_{23} = 0.127$ mm). The cylinders were mounted on a fixed insulating support located 30.5 cm above a horizontal ground plane. The separation between the cylinders was established by

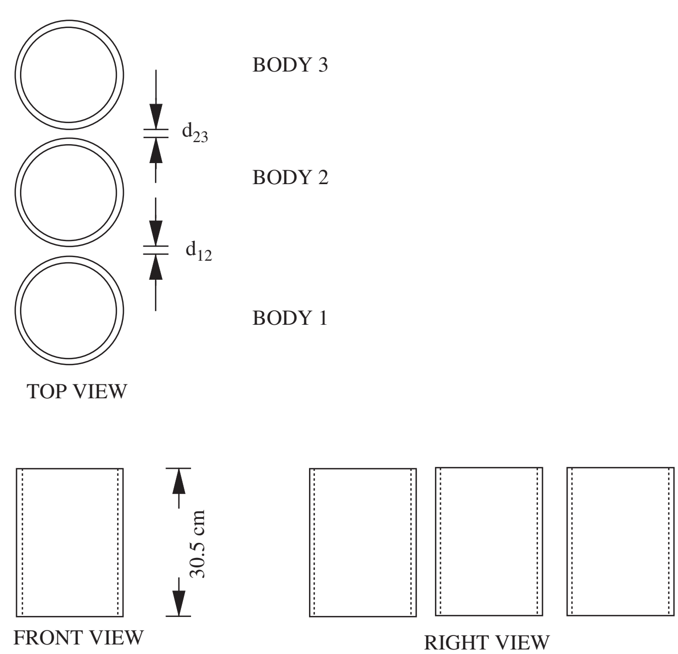

Fig. 2. Three cylinder test geometry: (M–M–M).