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Improved semi-analytic sensitivity analysis combined with a higher order approximation scheme in the framework of adjoint variable method

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Abstract

In the design sensitivity analysis, the adjoint variable method has been widely used for the sensitivity calculation. The adjoint variable method can reduce computation time and save computer resources because it can provide the sensitivity values only at the positions in which designers are willing to obtain. However, exact analytical differentiation with respect to the design variables is commonly employed in adjoint variable method. Although the exact derivative assures the accurate sensitivity, it is cumbersome to take differentiation in an exact manner for every given type of finite element. Therefore, in the present study, a new improved semi-analytic design sensitivity method is proposed in the framework of adjoint variable method. Recently, a numerical inaccuracy trouble in the traditional semi-analytic method has been settled by the rigid body mode separation technique and high order approximation scheme. Combining the adjoint variable method with improved semi-analytic design sensitivity scheme, the design sensitivity value can be calculated accurately and efficiently. Through numerical examples, the efficiency and accuracy of the proposed semi-analytic sensitivity scheme in the adjoint variable method are demonstrated.

Keywords: Design sensitivity; Adjoint variable method (AVM); Semi-analytical method (SAM); High order approximation scheme; Global finite difference method (GFDM); Rigid body mode

1. Introduction

Design optimization requires repeated calculations of design sensitivities in each iteration. There are several analytical methods for calculating sensitivity of structural response with respect to design parameters, including direct method and adjoint variable method (AVM). The direct method is classified into three categories, analytical method, global finite difference method (GFDM) and traditional semi-analytic method (TSAM). In these methods, TSAM is advantageous over others because TSAM compromises with the accuracy of the analytical method and the easiness of implementation of GFDM. But, if the unreasonable perturbation size is selected, numerical accuracy of the design sensitivity cannot be guaranteed. The direct method is not efficient in calculating sensitivities when the large number of design variables need to be considered because the direct method requires the corresponding sensitivity value for each design variable. AVM is more efficient when the number of design variables is larger than the number of displacement or stress constraints. In practical design situations we usually have to consider several load cases. In a multiple load-case simulation the adjoint method becomes more attractive. Therefore, AVM has been widely used in the calculation of sensitivity [1–6]. But, it is common to use the exact derivatives in AVM. Although AVM can provide the accurate sensitivity analysis, it is cumbersome to obtain the corresponding analytical design sensitivity for every type of finite element.

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Numerical derivative calculation by TSAM generates the truncation as well as round-off errors, which depend on the magnitude of perturbation size [7–10]. To overcome the inaccuracy trouble in TSAM, Van Keulen and De Boer have proposed the refined semi-analytic method (RSAM) [11,12]. RSAM is based on the rigid body mode separation and the exact differentiation of the rigid body modes. The RSAM eliminates the severe errors caused by the influence of the rigid body modes in the TSAM. Besides the reliability of the RSAM, the additional implementation effort of exact differentiation for rigid body modes is not heavy and it does not increase computing time. The reliable results by RSAM regardless of the perturbation size are shown in the references [11,13]. Recently, focusing semi-analytic sensitivity, various sensitivity approaches for linear static or dynamic analysis has been reviewed by Keulen [14].

The RSAM can be applied instead of using the exact analytical derivative of the stiffness matrix in AVM. Although the exact derivative in AVM assures the accurate sensitivity value, its implementation is not easy and require much time for evaluation of derivatives. Thus, the derivative calculation of the local stiffness matrix by RSAM assures the efficiency and the accuracy at the same time.

But, RSAM cannot improve the accuracy of TSAM, whose error comes from the truncation error when the perturbation sizes are large in the problem whose rigid body modes are not dominant and the pure deformation is significant. To improve this situation in the range of the large perturbation size, it is required to consider the higher order terms to eliminate the truncation error.

In this paper, the sensitivities of the displacement are calculated by AVM which is combined with RSAM and the higher order approximation method [15]. The higher order approximation method based on Von Neumann series expansion is combined with the mode decomposition technique to alleviate the truncation error as well as the round-off error [16]. This scheme works accurately and efficiently regardless of the perturbation size. The accuracy and efficiency of the present improved method are demonstrated for static problems.

Through the numerical examples, TSAM, GFDM, RSAM and higher order approximation method in arbitrary positions are evaluated by comparing the calculated sensitivities in the various range of the perturbation size.

2. Evaluation of design sensitivity in adjoint variable method

2.1. Adjoint variable method

The present sensitivity method is quite simple for the computer program implementation and the sensitivity computations do not depend on the details of element formulations.

For evaluating the sensitivity of the objective function $\mathbf{\Phi}$ by AVM, the adjoint variable $\boldsymbol{\lambda}$ should be obtained. In AVM, the sensitivity of the objective function is evaluated as Eq. (1).

$$\frac{\mathrm{d}\mathbf{\Phi}(\mathbf{u},b)}{\mathrm{d}b}=\frac{\partial\mathbf{\Phi}}{\partial b}+\frac{\partial\mathbf{\Phi}}{\partial\mathbf{u}}\frac{\partial\mathbf{u}}{\partial b} \tag{1}$$

where $\mathbf{u}$ is the displacement and b is the design variable. The objective function $\mathbf{\Phi}$ is dependent on $\mathbf{u}$ and b. The adjoint variable $\boldsymbol{\lambda}$ is defined in terms of the expressions given in Eq. (1).

$$\mathbf{K}(\mathbf{b})\boldsymbol{\lambda}=\frac{\partial\mathbf{\Phi}^{\mathrm{T}}}{\partial\mathbf{u}} \tag{2}$$

From the static equation, $\mathbf{Ku}=\mathbf{F}$, the displacement sensitivity by design variable b is evaluated as Eq. (3).

$$\frac{\partial\mathbf{u}}{\partial b}=\mathbf{K}^{-1}\left(\frac{\partial\mathbf{F}}{\partial b}-\frac{\partial\mathbf{K}}{\partial b}\mathbf{u}\right) \tag{3}$$

By inserting Eq. (3) into Eq. (1), Eq. (1) is evaluated as Eq. (4).

$$\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}b}=\frac{\partial\mathbf{\Phi}}{\partial b}+\frac{\partial\mathbf{\Phi}}{\partial\mathbf{u}}\mathbf{K}^{-1}\left(\frac{\partial\mathbf{F}}{\partial b}-\frac{\partial\mathbf{K}}{\partial b}\mathbf{u}\right) \tag{4}$$

Using the adjoint variable of the Eqs. (2), (4) is expressed in the following form.

$$\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}b}=\frac{\partial\mathbf{\Phi}}{\partial b}+\boldsymbol{\lambda}^{\mathrm{T}}\left(\frac{\partial\mathbf{F}}{\partial b}-\frac{\partial\mathbf{K}}{\partial b}\mathbf{u}\right) \tag{5}$$

In Eq. (5), the derivative of the stiffness matrix can be evaluated in an exact manner. But, although the exact differentiation of the stiffness matrix assures the accuracy, it is cumbersome to compute the corresponding derivative analytically for every given type of finite element. Semi-analytical sensitivity is much more efficient than the analytical sensitivity since it requires only numeric computation of derivatives of stiffness matrix. In this study, for assuring the reliable derivative of Eq. (5), RSAM is employed.

2.2. Evaluation of rigid body mode

In Fig. 1, the rigid body mode in element-level is extracted from force and moment equilibrium equation. The force equilibrium and moment equilibrium can be expressed as follows:

$$\begin{aligned}
&\mathbf{p}_1+\mathbf{p}_7+\mathbf{p}_{13}+\mathbf{p}_{19}=\mathbf{0}\\
&\mathbf{p}_2+\mathbf{p}_8+\mathbf{p}_{14}+\mathbf{p}_{20}=\mathbf{0}\\
&\mathbf{p}_3+\mathbf{p}_9+\mathbf{p}_{15}+\mathbf{p}_{21}=\mathbf{0}\\
&-\mathbf{p}_2\bar{\mathbf{z}}_1+\mathbf{p}_3\bar{\mathbf{y}}_1+\mathbf{p}_4-\mathbf{p}_8\bar{\mathbf{z}}_2+\mathbf{p}_9\bar{\mathbf{y}}_2+\mathbf{p}_{10}\\
&\quad-\mathbf{p}_{14}\bar{\mathbf{z}}_3+\mathbf{p}_{15}\bar{\mathbf{y}}_3+\mathbf{p}_{16}-\mathbf{p}_{20}\bar{\mathbf{z}}_4+\mathbf{p}_{21}\bar{\mathbf{y}}_4+\mathbf{p}_{22}=\mathbf{0}\\
&\mathbf{p}_1\bar{\mathbf{z}}_1-\mathbf{p}_3\bar{\mathbf{x}}_1+\mathbf{p}_5+\mathbf{p}_7\bar{\mathbf{z}}_2-\mathbf{p}_9\bar{\mathbf{x}}_2+\mathbf{p}_{11}\\
&\quad+\mathbf{p}_{13}\bar{\mathbf{z}}_3-\mathbf{p}_{15}\bar{\mathbf{x}}_3+\mathbf{p}_{17}-\mathbf{p}_{19}\bar{\mathbf{z}}_4+\mathbf{p}_{21}\bar{\mathbf{x}}_4+\mathbf{p}_{23}=\mathbf{0}\\
&-\mathbf{p}_1\bar{\mathbf{y}}_1+\mathbf{p}_2\bar{\mathbf{x}}_1+\mathbf{p}_6-\mathbf{p}_7\bar{\mathbf{y}}_2+\mathbf{p}_8\bar{\mathbf{x}}_2+\mathbf{p}_{12}\\
&\quad-\mathbf{p}_{13}\bar{\mathbf{y}}_3+\mathbf{p}_{14}\bar{\mathbf{x}}_3+\mathbf{p}_{18}-\mathbf{p}_{19}\bar{\mathbf{y}}_4+\mathbf{p}_{20}\bar{\mathbf{x}}_4+\mathbf{p}_{24}=\mathbf{0}
\end{aligned} \tag{6}$$