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Sensitivity analysis for generalized strongly monotone variational inclusions based on the (A, η)-resolvent operator technique

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Abstract

Sensitivity analysis for generalized strongly monotone variational inclusions based on the (A, η)-resolvent operator technique is investigated. The results obtained encompass a broad range of results.

Keywords: Sensitivity analysis; Quasivariational inclusions; Maximal relaxed monotone mapping; (A, η)-monotone mapping; Generalized resolvent operator technique

1. Introduction

Recently in [1] the author investigated sensitivity analysis for quasivariational inclusions using the resolvent operator technique. Resolvent operator techniques have been applied to a broad range of problems arising from several fields of research, especially from model equilibria problems in economics, optimization and control theory, operations research, transportation network modeling, and mathematical programming. In this work we present the sensitivity analysis for (A, η)-monotone quasivariational inclusions based on the generalized (A, η)-resolvent operator technique. The notion of (A, η)-monotone mappings upgrades the notion of A-monotonicity [2], which generalizes the well-known class of maximal monotone mappings to maximal relaxed monotone mappings. The results obtained generalize a wide range of results on the sensitivity analysis for quasivariational inclusions [3–6] and others. For more details, we recommend [1–8].

2. (A, η)-monotonicity

In this section we explore some basic properties derived from the notion of (A, η)-monotonicity. Let X denote a real Hilbert space with the norm $\| \cdot \|$ and inner product $\langle ., . \rangle$. Let $\eta : X \times X \to X$ be a single-valued mapping. The map η is called τ-Lipschitz continuous if there is a constant $\tau > 0$ such that

$$\|\eta(u, v)\| \leq \tau \|u - v\| \quad \forall u, v \in X.$$

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Definition 2.1. Let $\eta : X \times X \to X$ be a single-valued mapping, and let $M : X \to 2^X$ be a multivalued mapping on X. The map M is said to be:

(i) (r, η)-*strongly* monotone if

$$\langle u^* - v^*, \eta(u, v)\rangle \geq r\|u - v\|^2 \quad \forall (u, u^*), (v, v^*) \in \text{Graph}(M).$$

(ii) η-pseudomonotone if

$$\langle v^*, \eta(u, v)\rangle \geq 0$$

implies

$$\langle u^*, \eta(u, v)\rangle \geq 0 \quad \forall (u, u^*), (v, v^*) \in \text{Graph}(M).$$

(iii) (m, η)-relaxed monotone if there exists a positive constant m such that

$$\langle u^* - v^*, \eta(u, v)\rangle \geq (-m)\|u - v\|^2 \quad \forall (u, u^*), (v, v^*) \in \text{Graph}(M).$$

Definition 2.2. A mapping $M : X \to 2^X$ is said to be maximal (m, η)-relaxed monotone if

(i) M is (m, η)-relaxed monotone,
(ii) for $(u, u^*) \in X \times X$, and

$$\langle u^* - v^*, \eta(u, v)\rangle \geq (-m)\|u - v\|^2 \quad \forall (v, v^*) \in \text{Graph}(M),$$

we have $u^* \in M(u)$.

Definition 2.3. Let $A : X \to X$ and $\eta : X \times X \to X$ be two single-valued mappings. The map $M : X \to 2^X$ is said to be (A, η)-*monotone* if

(i) M is (m, η)-relaxed monotone,
(ii) $R(A + \rho M) = X$ for $\rho > 0$.

Note that alternatively, the map $M : X \to 2^X$ is said to be (A, η)-monotone if

(i) M is (m, η)-*relaxed* monotone,
(ii) $A + \rho M$ is η-pseudomonotone for $\rho > 0$.

Proposition 2.1. *Let $A : X \to X$ be an (r, η)-strongly monotone single-valued mapping, and let $M : X \to 2^X$ be an (A, η)-monotone mapping. Let $\eta : X \times X \to X$ be τ-Lipschitz continuous single-valued mapping. Then M is maximal (m, η)-relaxed monotone for* $0 < \rho < \frac{r}{m}$.

Proposition 2.2. *Let $A : X \to X$ be an (r, η)-strongly monotone single-valued mapping, and let $M : X \to 2^X$ be an (A, η)-monotone mapping. In addition, let $\eta : X \times X \to X$ be τ-Lipschitz continuous. Then $(A + \rho M)$ is maximal η-monotone for* $0 < \rho < \frac{r}{m}$.

Proposition 2.3. *Let $A : X \to X$ be an (r, η)-strongly monotone mapping, and let $M : X \to 2^X$ be an (A, η)-monotone mapping. If, in addition, $\eta : X \times X \to X$ is τ-Lipschitz continuous, then the operator $(A + \rho M)^{-1}$ is single-valued for* $0 < \rho < \frac{r}{m}$.

Definition 2.4. Let $A : X \to X$ be an (r, η)-strongly monotone mapping, and let $M : X \to 2^X$ be an (A, η)-monotone mapping. Then the generalized resolvent operator $J^M_{\rho,A} : X \to X$ is defined by

$$J^M_{\rho,A}(u) = (A + \rho M)^{-1}(u).$$

Definition 2.5. The map $T : X \times X \times L \to X$ is said to be r-*strongly* monotone with respect to A in the first argument if there exists a positive constant r such that

$$\langle T(x, u, \lambda) - T(y, u, \lambda), A(x) - A(y)\rangle \geq (r)\|x - y\|^2 \quad \forall (x, y, u, \lambda) \in X \times X \times X \times L.$$