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Design sensitivity analysis for parameters affecting geometry, elastic–viscoplastic material constant and boundary condition by consistent tangent operator-based boundary element method

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Abstract

This paper presents a design sensitivity analysis method by the consistent tangent operator concept-based boundary element implicit algorithm. The design variables for sensitivity analysis include geometry parameters, elastic–viscoplastic material parameters and boundary condition parameters. Based on small strain theory, Perzyna's elastic–viscoplastic material constitutive relation with a mixed hardening model and two flow functions is considered in the sensitivity analysis. The related elastic–viscoplastic radial return algorithm and the formula of elastic–viscoplastic consistent tangent operator are derived and discussed. Based on the direct differentiation approach, the incremental boundary integral equations and related algorithms for both geometric and elastic–viscoplastic sensitivity analysis are developed. A 2D boundary element program for geometry sensitivity, elastic–viscoplastic material constant sensitivity and boundary condition sensitivity has been developed. Comparison and discussion with the results of this paper, analytical solution and finite element code ANSYS for four plane strain numerical examples are presented finally.

Keywords: Geometry/material sensitivity; Boundary element method; Elastic–viscoplasticity; Consistent tangent operator

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1. Introduction

Sensitivity analysis of non-linear (material and/or geometrical) problems plays an important role in structural optimization, inverse problem and reliability analysis. Both finite element method (FEM) (Arora and Cardoso, 1992; Jao and Arora, 1992a,b; Choi and Santos, 1987; Santos and Choi, 1988; Badrinarayanan and Zabaras, 1996; Rojc and Tok, 2003; Kim and Choi, 2001; Cho and Lee, 2002) and continuum boundary

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element method (BEM) (Mukherjee and Chandra, 1989, 1991; Zhang and Mukherjee, 1992; Zhang et al., 1992; Wei et al., 1994; Leu and Mukherjee, 1994a,b, 1995) have been developed for geometry and material non-linear sensitivity analysis by a lot of researchers. Currently, there are three different approaches that are used in sensitivity analysis: the finite difference approach (FDA), the adjoint structure approach (ASA), and the direct differentiation approach (DDA). Among these three differentiation approaches, ASA, similarly as DDA, consists in exact analytical differentiation of primary equations, and for large number of design parameters it is advocated as more efficient than DDA (Haug et al., 1986). However for the non-linear history-dependent problems, the DDA has been seen to be more suitable (Tsay and Arora, 1989; Kleiber et al., 1997). Note that for non-linear problems, an incremental-iterative numerical method is needed. Therefore, a powerful and high efficiency algorithm for non-linear solver is the cornerstone of a successful non-linear analysis. The concept of consistent tangent operator (CTO), which is first proposed in finite element method by Simo and Taylor (1985), has obtained wide application in sensitivity analysis of non-linear problems. Use of the CTO, as it was pointed out by Vidal et al. (1991), Vidal and Haber (1993), Kleiber and Hien (1991), Kleiber et al. (1994, 1995) and Michaleris et al. (1994), provides very accurate numerical results in sensitivity analysis; while other approaches (e.g. using the continuum tangent operator) might lead to significant errors. Bonnet and Mukherjee (1996), for the first time, have introduced the CTO concept in boundary element and small strain elastic plastic sensitivity analysis. Later, Poon et al. (1998) have further developed this method into 2D elastoplastic sensitivity problem. However, in these papers of CTO-based BEM (Bonnet and Mukherjee, 1996; Poon et al., 1998), only elastic plastic material sensitivity parameter is studied. The viscoplastic material sensitivity, geometry sensitivity and boundary condition sensitivity analysis are not considered. Recently, Liang et al. (2004) have solved the viscoplastic material sensitivity problem with CTO-based implicit BEM, but the geometry sensitivity and boundary condition sensitivity have not yet been developed.

Therefore, the goal of this paper is to present a sensitivity analysis method for parameters affecting geometry, elastic–viscoplastic material constant and boundary condition with the CTO-based small strain boundary element. The CTO plays a pivotal role in the present work. The design variables for sensitivity analysis include geometry (shape, dimension and size) parameters, elastic–viscoplastic material parameters and boundary condition parameters. The organization of the paper is arranged as follows: First, based on small strain theory, Perzyna's elastic–viscoplastic constitutive relation is introduced with the mixed strain-hardening material that includes both isotropic and kinematic cases. Two types of viscoplastic flow functions with exponent-type and power-type are built in the viscoplastic material constitutive relation. Secondly, the elastic–viscoplastic CTO-based boundary element and related radial return algorithm (RRA) are derived with new formulae of RRA and CTO which combine the mixed strain-hardening model and both exponent type and power-type of the flow functions. Then, based on the direct differentiation approach, the fully incremental boundary integral equations of geometry sensitivity, elastic–viscoplastic sensitivity and boundary condition sensitivity are developed together with the new sensitivity formulation of RRA and CTO-based equations. A non-linear algorithm for geometry, elastic–viscoplastic material and boundary condition sensitivities is developed. Finally, four plane strain numerical examples with geometry sensitivity, elastic–viscoplastic material constant sensitivity and boundary condition sensitivity analysis are presented and discussed.

2. Elastic–viscoplastic model

In classical formulations of elastic–viscoplasticity, the yield criterion is defined through a loading function $F \equiv F(\boldsymbol{\sigma}, \mathbf{q})$, where $\boldsymbol{\sigma}$ denotes the stress state and $\mathbf{q}$ denotes the internal variables. As elastic–viscoplastic deformation appears, the stress is permissible outside the closure of the loading surface, *i.e.* $F(\boldsymbol{\sigma}, \mathbf{q}) > 0$. However, in rate-independent plasticity, $F(\boldsymbol{\sigma}, \mathbf{q}) \leqslant 0$, it is the basic difference between viscoplasticity and rate-independent plasticity.

For the classic elastic–viscoplastic constitutive model (see Fig. 1, where σ_s is the yield stress), the total strain rate is sum of its elastic and viscoplastic components,

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{\mathrm{e}} + \dot{\boldsymbol{\varepsilon}}^{\mathrm{vp}} \tag{1}$$

where the superscript "e" indicate the elastic component and the superscript "vp" indicate the viscoplastic component.