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Reliability and sensitivity analysis of a system with multiple unreliable service stations and standby switching failures

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Abstract

This paper presents the reliability and sensitivity analysis of a system with M primary units, W warm standby units, and R unreliable service stations where warm standby units switching to the primary state might fail. Failure times of primary and warm standby units are assumed to have exponential distributions, and service times of the failed units are exponentially distributed. In addition, breakdown times and repair times of the service stations also follow exponential distributions. Expressions for system reliability,$R_Y(t)$, and mean time to system failure, $MTTF$ are derived. Sensitivity analysis, relative sensitivity analysis of the system reliability and the mean time to failure, with respect to system parameters are also investigated.

Keywords: Mean time to system failure; Reliability; Sensitivity analysis; Switching failure

1. Introduction

In machine repair problems, most studies about the reliability of a system assume that the switchover from warm standby units to primary units is always perfect and the service stations are reliable. Although these assumptions simplify the analysis of the problem, they might not reflect certain real situations. For instance, a warm standby unit with a lower failure rate might not be able to switch over to a primary unit successfully, and it might also need a longer warm-up time. In this paper, we investigate the reliability of a system in which the switching failure might occur and service stations might also break down. Although the concept of imperfect switching was discussed by Lewis [1], to the best of our knowledge, it has never been considered in the reliability problem in a system with multiple unreliable service stations.

Under the assumption of perfect switching, Wang [2] proposed the M/M/1 machine repair problem with two different types of a single service station subject to breakdowns. Later, Wang and Kuo [3] extended the result to the M/E_k/1 machine repair problem with a single unreliable server. Wang [4] also developed steady-state analytic solutions for the M/M/R machine repair problem with spares and R unreliable service

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stations. Cao [5] derived reliability measures of a machine service model with a single unreliable service station. Wang and Sivazlian [6] presented the reliability characteristics of a repairable system with warm standbys. Ke and Wang [7,8] analyzed the reliability characteristics of a repairable system with warm standby units in which failed units balk with a constant probability and renege according to an exponential distribution. Lately, Wang et al. [9] analyzed the reliability and sensitivity of a repairable system with warm standbys and multiple unreliable service stations.

On the other hand, the reliability of the system with a switching device which might fail was also discussed. Chung [10] gave the reliability and availability of k operating machines and s cold standbys with multiple repair facilities and multiple critical and non-critical errors when the switching mechanism is subject to failure. Gurov and Utkin [11] presented the transient behavior of repairable and unrepairable cold standby systems with conversion switches. They established mathematical models of systems by a set of integral equations. Pan [12] discussed a non-repairable system with one or two standby components, one switch, and one sensor. Coit [13] derived a solution methodology to determine optimal design configurations for non-repairable systems with cold standbys, non-constant hazard functions, and imperfect switching.

In this paper, we examine the reliability characteristics of a system with M identical primary units operating simultaneously in parallel, W warm standby units, and R unreliable service stations. This model is an extension of Wang et al. [9] model with the consideration of imperfect switching. In particular, we study the impact of the switching failure on the reliability function and the mean time to failure. In addition, the sensitivity and the relative sensitivity of the system reliability as well as of the mean time to failure with respect to system parameters are investigated.

The rest of the paper is organized as follows. In Section 2, we formulate the problem and provide notation subsequently used throughout the paper. In Section 3, explicit expressions for reliability function, $R_Y(t)$, and the mean time to system failure, the $MTTF$ are derived using Laplace transform techniques. Sensitivity and relative sensitivity analysis of $R_Y(t)$ and the $MTTF$ are also developed in terms of the system parameters. In Section 4, numerical results are provided to illustrate the sensitivity and the relative sensitivity of $R_Y(t)$ and of the $MTTF$ with respect to system parameters. Conclusions are presented in the last section.

2. Problem formulation and notation

In this paper, we consider a system which consists of M identical primary units operating simultaneously in parallel, W warm standby units and R unreliable service stations.

The assumptions of the model are described as follows. Suppose that the failures of the primary units and those of the warm standbys occur independently of the states of other units and follow exponential distributions with parameters λ and α (where $0<\alpha<\lambda$), respectively. When a primary unit fails, it is immediately replaced by a warm standby if one is available. It is assumed that the switchover time is instantaneous. However, the switch to a primary unit is imperfect with a failure probability q. If a warm standby unit fails to switch to a primary unit, the next available standby unit attempts to switch. This process continues until switching is successful or all the warm standby units have failed. When a warm standby unit switches over successfully, its failure characteristics become those of a primary unit. If a primary or a warm standby unit fails, it is immediately sent to the service station. In addition, the time to serve a failed unit is exponentially distributed with parameter μ. Once a unit is repaired, it instantly resumes warm standby status. Moreover, we assume that the service station can break down at any time with breakdown rate α. Whenever the service station breaks down, it is immediately repaired with repair rate β. Again, breakdown times and repair times of the service station are assumed to be exponentially distributed. It is assumed that each service station can serve only one failed unit at a time and that service is independent of unit failures. If all service stations fail, then failed units must wait until a service station is repaired. If the service of a failed unit is interrupted by a breakdown, the service resumes as soon as another service station is available or the repair completion terminates. Once a service station is repaired, it becomes as good as new and immediately serves the first failed unit in the queue. Although no service occurs during the repair period if all service stations have failed, failed units continue to arrive according to a Poisson process.