Available online at www.sciencedirect.com
ScienceDirect
Int. J. Production Economics 108 (2007) 302–313
international journal of production economics
www.elsevier.com/locate/ijpe

Global sensitivity analysis in inventory management

E. Borgonovo*, L. Peccati

IMQ, Bocconi University, Viale Isonzo 25, 20135 Milano, Italy

Available online 18 January 2007

Abstract

This paper deals with the sensitivity analysis (SA) of inventory management models when uncertainty in the input parameters is given full consideration. We make use of Sobol' function and variance decomposition method for determining the most influential parameters on the model output. We first illustrate the method by means of an analytical example. We provide the expression of the global importance of demand, holding costs, order costs of the Harris economic order quantity (EOQ) formula. We then present the global SA of the inventory management model developed by Luciano and Peccati [1999. Capital structure and inventory management: the temporary sale problem. International Journal of Production Economics 59, 169–178] for the economic order quantity estimation in the context of the temporary sale problem. We show that by performing global SA in parallel to the modeling process an analyst derives insights not only on the EOQ structure when its expression is not analytically known, but also on the relevance of modeling choices, as the inclusion of financing policies and special orders.

Keywords: Inventory management; Parameter uncertainty; Global sensitivity analysis; EOQ

1. Introduction

Uncertainty in inventory policy making stems from a variety of factors. Just as a simple example, consider a firm that uses the Harris economic order quantity (EOQ) formula as a support to its inventory policies (Erlenkotter, 1990; Harris, 1915; Piasecki, 2001). In order to come to a final decision on the EOQ, the firm must estimate demand, unit order costs and holding costs. Demand is seldom steady, and its value cannot be determined with certainty in most of the cases (Alstrom, 2001; Ray and Chaudhuri, 1997; Grubbstrõm, 1996; Teng and Yang, 2004; Matheus and Gelders, 2000; Boylan and Johnston, 1996). Costs can be a further source of uncertainty (see Piasecki, 2001): on the one hand the criteria of cost classification are not always be sharply set and, on the other hand, even once the criteria are set, variability characterizes the costs themselves (Piasecki, 2001). Hence, rarely one can predict the behavior of an inventory system with the inputs fixed at a certain value; more likely, the decision-maker will be able to assign parameters within ranges determined by the analysis (Piasecki, 2001; Bogataj, 1998; Bogataj and Hvalica, 2003). To cope with the corresponding uncertainty in model predictions, usually a sensitivity analysis (SA) exercise is performed. The more direct SA scheme is the testing of the change in model output that follows a change in the parameters when they are

*Corresponding author.
E-mail address: emanuele.borgonovo@uni-bocconi.it (E. Borgonovo).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2006.12.027

shifted within the limits of their variation ranges: this type of one-variable-at-a-time SA is performed in Ray and Sahu (1992), Arcelus and Rowcroft (1993), Ray and Chaudhuri (1997), and Powell (2000). Ganeshan et al. (2001) study the sensitivity of supply chain performance to three inventory parameters. Perturbation analysis has been developed and employed in the works of Glasserman and Tayur (1995), Bogataj and Bogataj (1992), Cibej and Bogataj (1994), Bogataj and Cibej (1994), and Bogataj and Bogataj (2004). The previous approaches belong to the family of local SA approaches. Local SA techniques are the set of methods that study the behavior of a mathematical model around a point of the input parameter space for finite or small changes in the input parameters (Borgonovo et al., 2003). To the family of local SA methods belong the technique of comparative statics (Samuelson, 1947) and the differential importance measure (Borgonovo and Apostolakis, 2001; Borgonovo and Peccati, 2004). An SA method is global if it tests the sensitivity of the model in consideration of the uncertainty distribution reflecting the decision-maker state of knowledge in the parameters. Global SA techniques can be divided in the categories of non-parametric techniques (Saltelli and Marivoet, 1990), screening methods (Morris, 1991), response surface methodology (Downing et al., 1985), and variance-based methods (Saltelli et al., 1999, 2000b; Sobol', 1993). Target of variance-based global SA methods is the model output variance, which is decomposed in a series of summands of increasing dimensionality (Saltelli et al., 1999; Sobol', 1993, 2003; Saltelli et al., 2000b). In this work, we focus on variance decomposition through the Sobol' (Saltelli et al., 2000b; Sobol', 1993) and the extended FAST methods (Saltelli et al., 1999).

Our first application is the determination of the global importance of the parameters in the classical Harris inventory management model (Harris, 1915; Erlenkotter, 1990). We derive analytically Sobol' function and variance decomposition of the Harris EOQ formula and provide the expression of the input parameter global importance (GI). By means of numerical results, we illustrate that parameters associated with the highest value of GI are the most effective in reducing the variance of the EOQ.

We then apply the techniques to the inventory management model introduced by Luciano and Peccati (1999) in the context of the temporary sale problem. Starting point of the model is the loss function corresponding to the consideration of the cost of capital which is obtained by making use of the adjusted present value (APV) technique (Grubbström and Ashcroft, 1991; Grubbström and Thorstenson, 1986; Peccati, 1989, 1996; Thorstenson, 1988). Luciano and Peccati (1999) then formulate the cost functions that include third party financing, and the presence of special orders. None of the cost functions allows for the analytical expression of the EOQ. We show that performing a step-by-step global SA an analyst gains insights on both the modeling aspects and the EOQ structure. As far as modeling aspects are concerned, the use of global SA allows to ascertain whether the inclusion/exclusion of a certain assumption has a significant or negligible impact on the EOQ. As far as the EOQ structure is concerned, one gains a quantitative indication on the type of dependency of the EOQ on the parameters. This information would not be gained without a global SA when one does not possess the analytical expression of the EOQ. Results show that, for the uncertainty ranges at hand, the cost of debt, followed by the cost of capital and the special order discount are the most relevant parameters. Not only, but their inclusion impacts the EOQ structure in a significant way, shifting the EOQ dependence on the parameters from additive to non-additive.

Section 2 describes the principles and theorems at the basis of global SA. Section 3 illustrates the application and provides analytical results for the Harris EOQ formula. Section 4 presents the global SA of the Luciano–Peccati model, comparing the different cost functions and the corresponding SA results. Conclusions are offered in Section 5.

2. Variance-based global SA

The purposes of performing a global SA of model output are many, as the works of Saltelli (1999) and Saltelli et al. (2000b) discuss. Two are the characteristics of global SA that we are going to exploit and discuss in this work:

- the ability to enable the understanding of the type of model structural dependence on the input parameters when the explicit dependence is not available;
- the ability to assess the influence of parameters in consideration of the decision-maker state of knowledge, thus providing guidance in data collection